CAREER & COLLEGE ready, set, go!

ACRE Accountability and Curriculum Reform Effort in Response to *A Framework For Change*

PUBLIC SCHOOLS OF NORTH CAROLINA State Board of Education | Department of Public Instruction

Common Core State Standards K-5 Mathematics

Presented by

NC Department of Public Instruction

Introductions

Microsoft

Parking Lot

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Breaks

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Technology

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Session Materials

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Norms

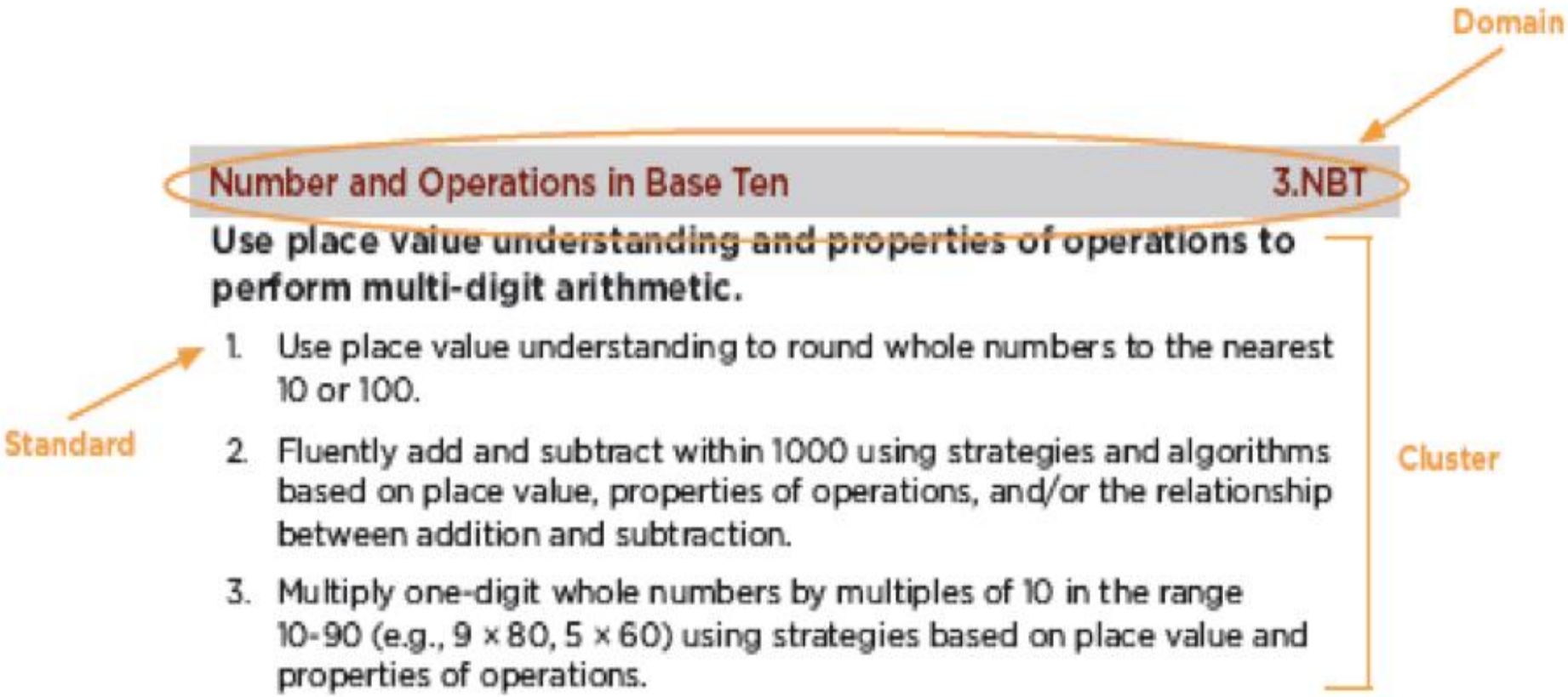
- Listen as an Ally
- Value Differences
- Maintain Professionalism
- Participate Actively

Timeline for Common Core Mathematics Implementation

Common Core State Standards Adopted June, 2010

Year	Standards To Be Taught	Standards To Be Assessed
2010 – 2011	2003 NCSCOS	2003 NCSCOS
2011 – 2012	2003 NCSCOS	2003 NCSCOS
2012 – 2013	CCSS	CCSS

Reading the Grade Level Standards



Sample Crosswalk

NC SCOS			Common Core			
Strand	Objective	Text of objective	Domain	Standard	Cluster Text of objective	Comments
Numbers & Operations	1.01	Develop number sense for whole numbers through 9,999. a) Connect model, number word, and number using a variety of representations. b) Build understanding of place value (ones through thousands). c) Compare and order.	Numbers & Operations in Base Ten	3.NBT.1	**Use place value understanding and properties of operations to perform multi-digit arithmetic.** Use place value understanding to round whole numbers to the nearest 10 or 100.	Builds on the understanding of place value in order to round numbers; however the majority of the 2003 objective will be taught prior to 3rd grade in Common Core.
	1.02	Develop fluency with multi-digit addition and subtraction through 9,999 using: a) Strategies for adding and subtracting numbers. b) Estimation of sums and differences in appropriate situations. c) Relationships between operations.	& Operations in Base Ten	3.NBT.2	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**[4] Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	[4]A range of algorithms may be used. Students should engage in using various strategies such as decomposing and composing numbers (e.g., 259 + 145 = 200 + 100 + 50 + 40 + 9 +5), etc. to solve problems.

http://www.dpi.state.nc.us/acre/standards/support-tools/

CAUTION!!

CONTENT APPEARING TO BE THE SAME MAY ACTUALLY BE DIFFERENT!!

The CCSS Requires CLOSE Reading!!!

Instructional Support Tools

Unpacked Content

- **A response, for each standard, to the question "What does this standard mean?"**
- The unpacked content is text that describes carefully and specifically what the standards mean a child will now, understand and be able to do and explains the different knowledge or skills that constitute that standard.

Sample of Unpacking

Operations and Algebraic Thinking	3.0A
Common Core Cluster	
Represent and solve problems involving multiplication and division.	
Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.	
Common Core Standard	**Unpacking** What do these standards mean a child will know and be able to do?
3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	**3.OA.1** Interpret products of whole numbers. Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3.
3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	**3.OA.2** focuses on two distinct models of division: partition models and measurement (repeated subtraction) models. Partition models focus on the question, "How many in each group?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag? OOOO OOOO OOOO Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? OOO OOO OOO OOO
3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number	**3.OA.3** students should apply their skills to solve word problems. This could include one or two-step word problems, such as: If you divide 4 dozen brownies among 8 people, how many cookies does each person receive? *(4x12 = 48, 48 ÷ 8 = 6).* Glossary, Table 2 gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the

http://www.dpi.state.nc.us/acre/standards/support-tools/

Our Goals for Today

- ✓ Understand certain **critical ideas** from CCSS
- ✓ Recognize CCSS as **Fewer, Clearer, Higher**
- ✓ Recognize how **Standards for Practice** mandate better ways of managing instruction
- ✓ Make **connections** between CCSS and Teacher Evaluation Tool

ALGEBRA

Algebra is a generalization of arithmetic.

Marilyn Burns

“If there is a problem with algebra in your high schools, then you have to fix it in K-4.”

Kathy Richardson, NCTM, 2004

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.3. **Decompose numbers less than or equal to 10 into pairs in more than one way**, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

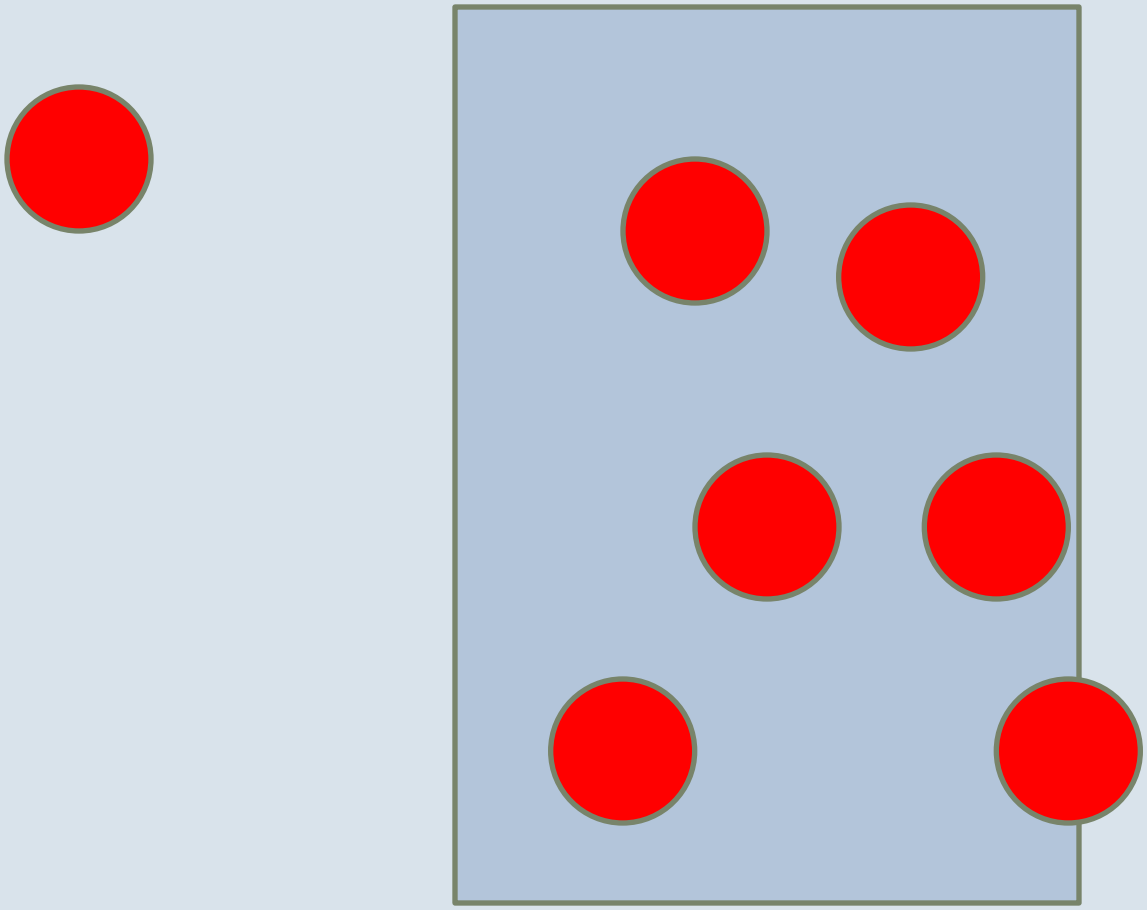
NOTE: Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.

(page 11)

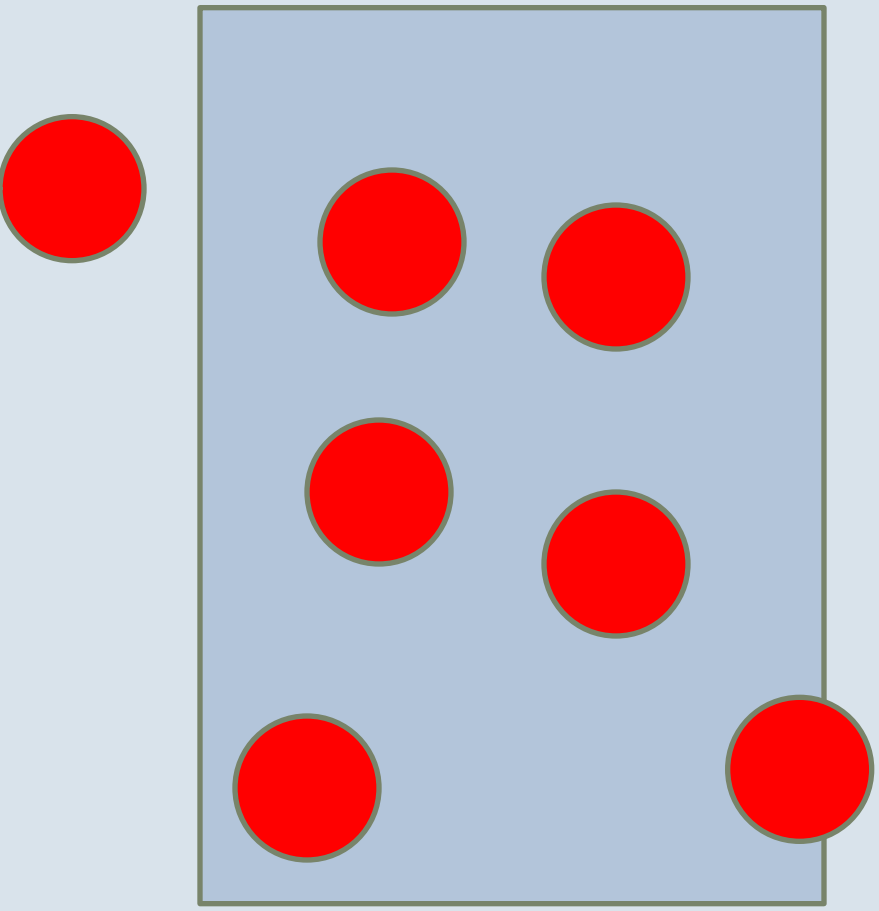
“On and Off”

- Let’s work on the number 7.
- Drop 7 counters over a piece of paper.
- How many counters land on the paper? off the paper?
- Record your work.
- Repeat 12 times.

On and Off



On and Off



On	Off
6	1

Turn and Talk

- What did you notice?
- How many ways could you make 7?
- Did some seem alike? different?
- Did you always have 7 no matter how many were on or off?

Turn and Talk

- Does this task have multiple entry points?
- How can it be differentiated?
- Will it help ELL students build understanding?

Suggested support For ELLs

- Sentence frames
  - ❑ There are ___ counters **on** the paper.
  - ❑ There are ___ counter **off** the paper.

  English language proficiency level (expanding bridging)
  - ❑ ____counters landed on the paper.
  - ❑ ____ counters landed off the paper.

$$8 + 4 = [\ \ ] + 5$$

8 + 4 = [ ] + 5

	Percent Responding with Answers			
Grade	**7**	**12**	**17**	**12 & 17**
1st - 2nd				
3rd - 4th				
5th - 6th				

Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.
Carpenter, Franke, & Levi
Heinemann, 2003

8 + 4 = [ ] + 5

	Percent Responding with Answers			
Grade	**7**	**12**	**17**	**12 & 17**
1st - 2nd	5	58	13	8
3rd - 4th				
5th - 6th				

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8 + 4 = [ ] + 5

	Percent Responding with Answers			
Grade	**7**	**12**	**17**	**12 & 17**
1st - 2nd	5	58	13	8
3rd - 4th	9	49	25	10
5th - 6th				

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8 + 4 = [ ] + 5

	Percent Responding with Answers			
Grade	**7**	**12**	**17**	**12 & 17**
1st - 2nd	5	58	13	8
3rd - 4th	9	49	25	10
5th - 6th	2	76	21	2

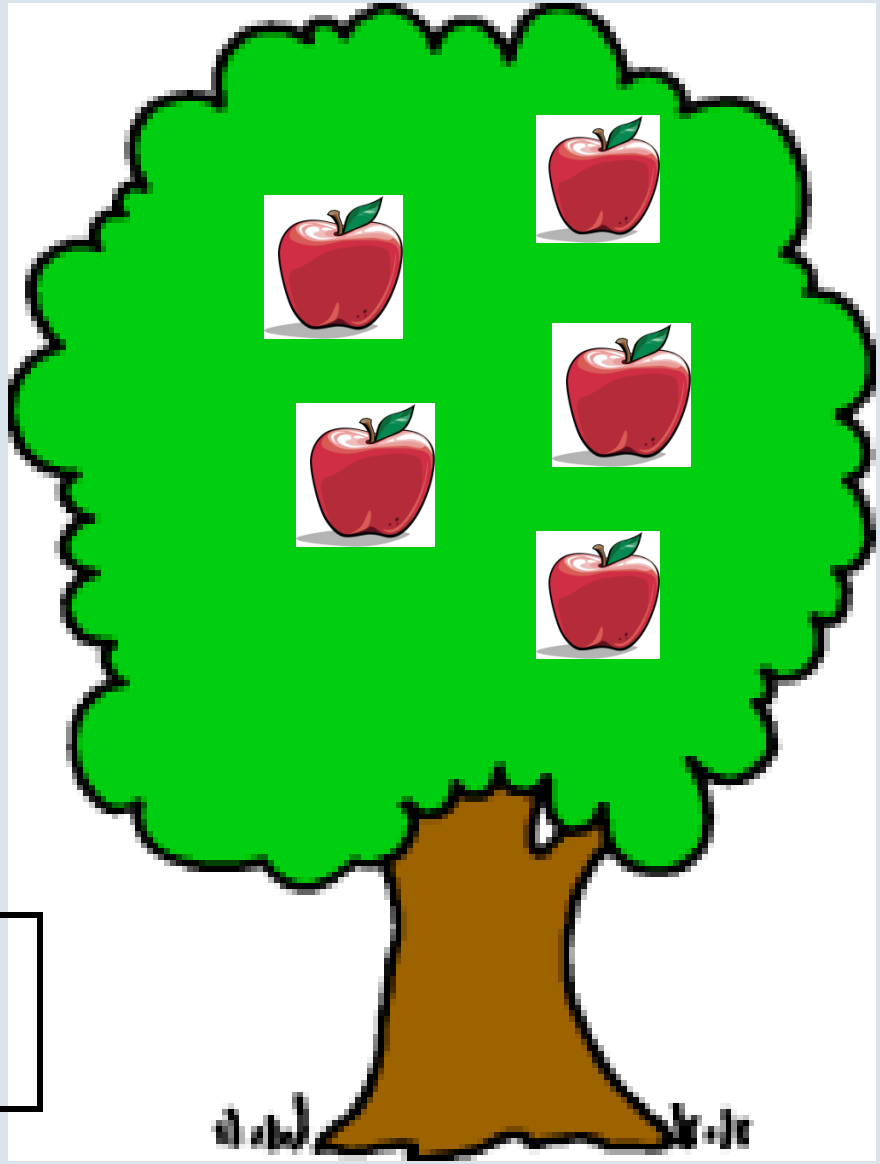
Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.
Carpenter, Franke, & Levi
Heinemann, 2003

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 **Represent addition and subtraction** with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

NOTE: Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.

(page 11)



$5 - 3 = 2$

Work with addition and subtraction equations.

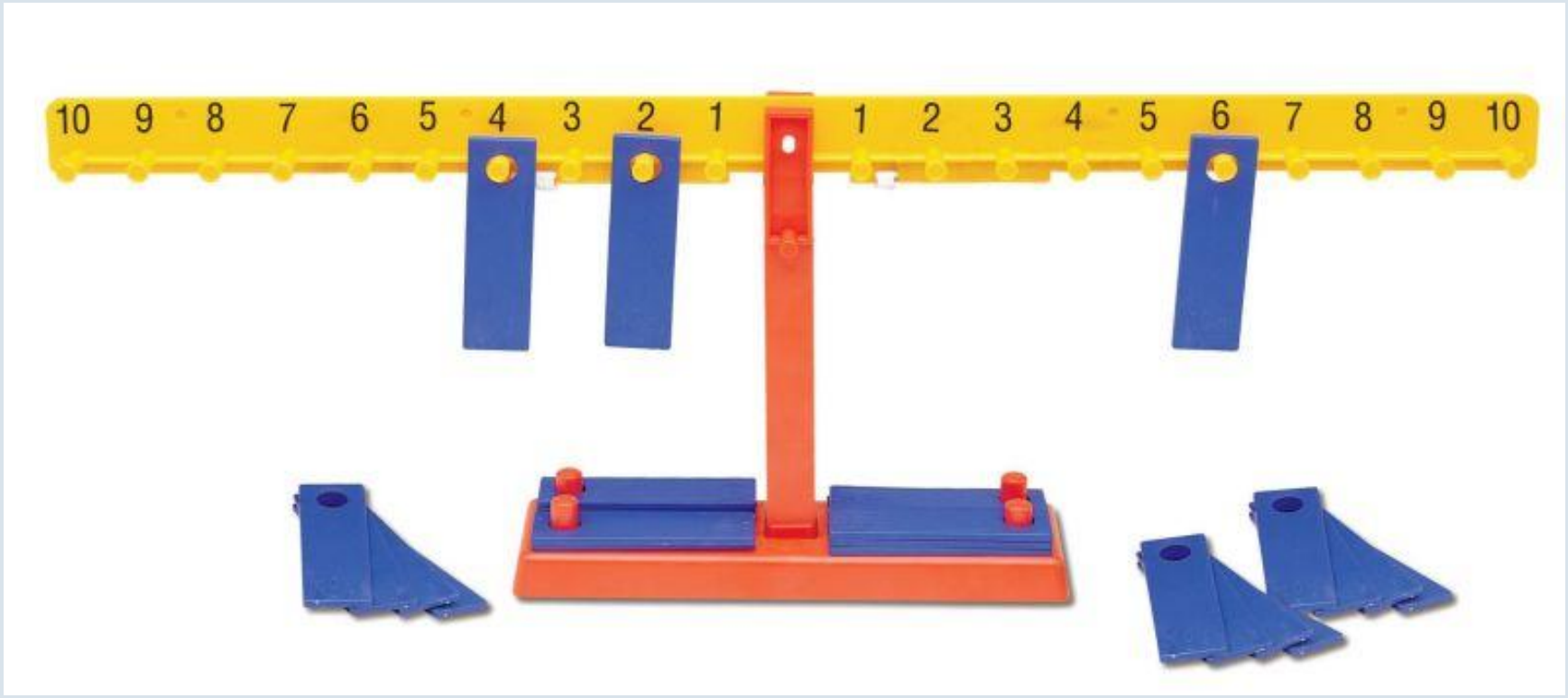
1.OA.7 **Understand the meaning of the equal sign**, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

Think of 8 + 4 = __ + 5.

Building on tasks like On and Off, students can think about how

3 + 4 = 5 + 2 or 5 + 2 = 6 + 1

Number Balance



Represent and solve problems involving addition and subtraction.

2.OA.1. **Use addition and subtraction within 100** to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, **with unknowns in all positions**, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

Grade Two

Students use context problems to learn about a variety of problem structures.

- **Sneha has 9 apples. Kitty gives her 4 more. How many does she have? 9 + 4=__**

- **Sneha has some apples. Kitty gives her 4 more. Now she has 11 apples. How many did she have to begin with? __ + 4 = 11**

Common Core Glossary

Table 1. Common addition and subtraction situations

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1

Grade Three

- **Understanding different problem structures moves students to multiplication and its relationship to division.**

- **Table 2 on page 89**

Common Core Glossary

Table 2. Common multiplication and division situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Third Grade Critical Area

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models***; multiplication is finding an unknown product, and division is finding an unknown factor in these situations.*** For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

There are 4 space ships. Each spaceship has 3 durdles (legs). There are 12 durdles all together.

Write three riddles for this story with three different answers.
(Put an unknown in each position. Table 2, p. 89)

Work with addition and subtraction equations.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, **including problems in which remainders must be interpreted**. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

48 ÷ 5 =

a) 10

b) 9 r 3

c) 9 3/5

d) 9.60

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7)*. Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Which of the following could be an estimate for the number represented by this expression: 3(18933 + 921)

a) 19854
b) 60000
c) 319854
d) 30,000

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7).* ***Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.***

Which of the following could be an estimate for the number represented by this expression: 3(18933 + 921)

a) 19854
b) 60000
c) 319854
d) 30,000

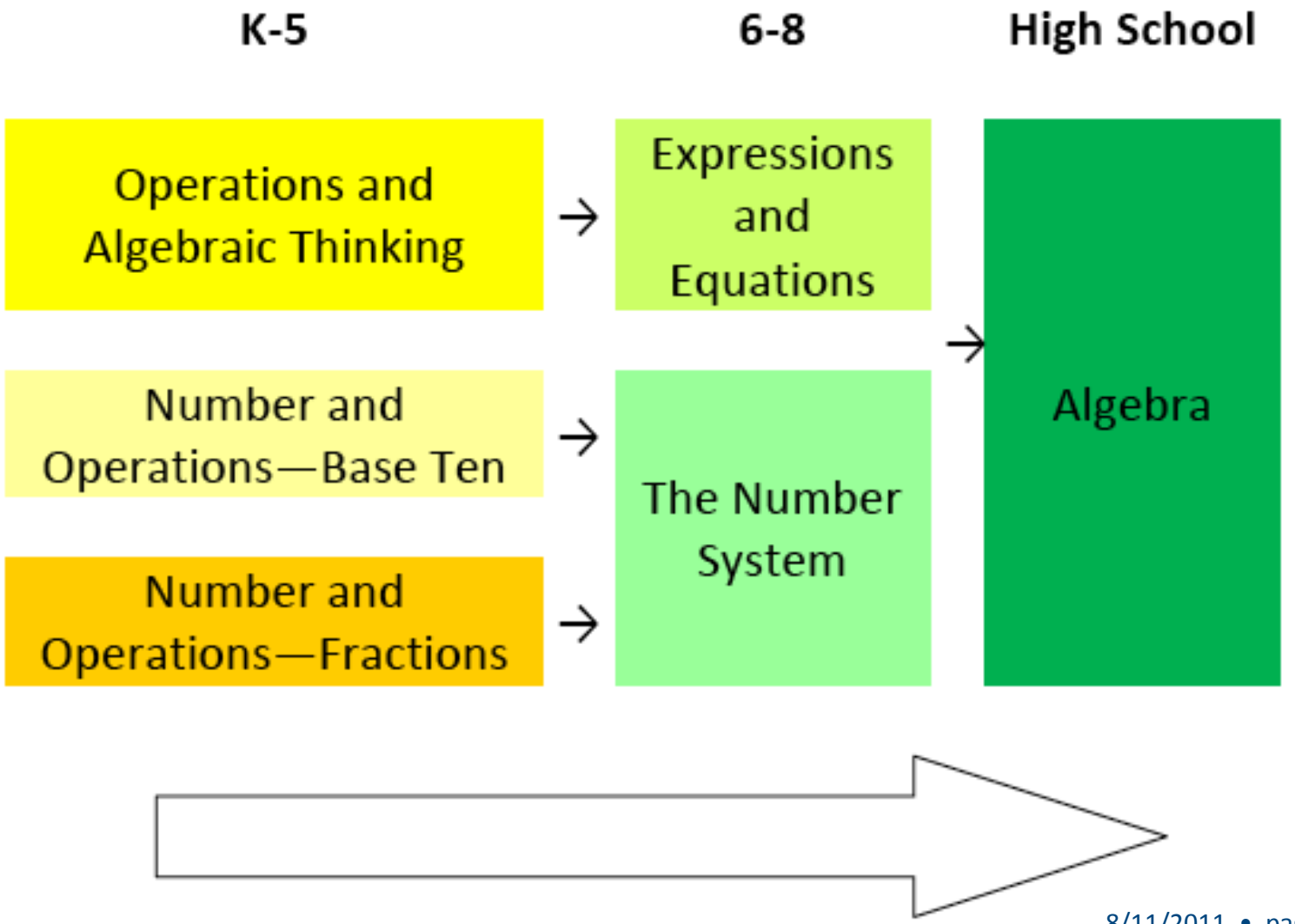
Exploring Algebra

- **Work with a partner.**
- **Look at one or more grade levels of the Operations and Algebraic Thinking Standards.**

“Arithmetic is a rehearsal for algebra.”

Bill McCullam, CCSS Mathematics Author

Focusing attention within Number and Operations



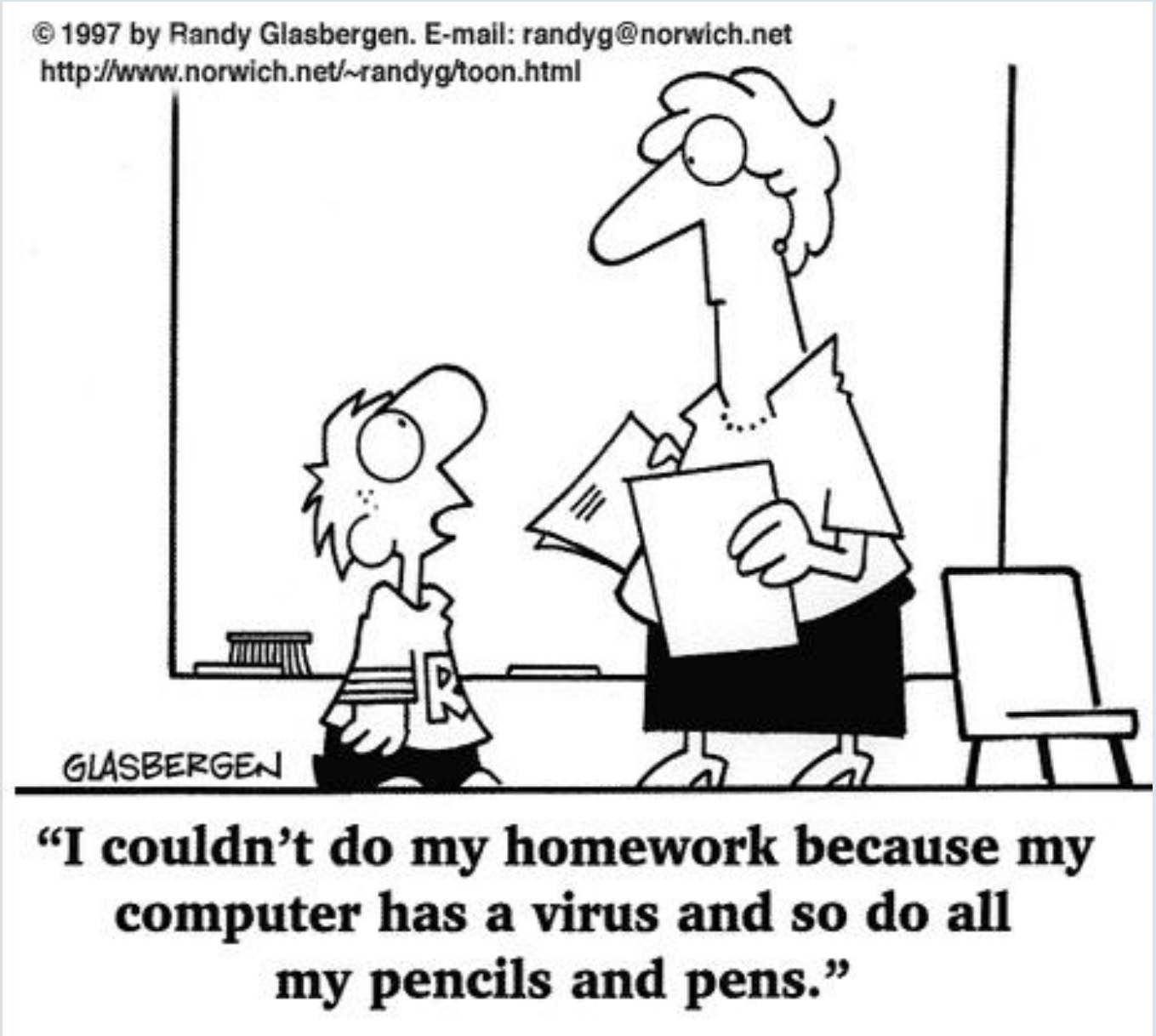
Old Boxes

- People are the next step
- If people just swap out the old standards and put the new CCSS in the old boxes
  - into old systems and procedures
  - into the old relationships
  - Into old instructional materials formats
  - Into old assessment tools,
- Then nothing will change, and perhaps nothing will

Phil Daro, NCCTM 2010

Time to Reflect

Summary	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GLASBERGEN
"I couldn't do my homework because my computer has a virus and so do all my pencils and pens."

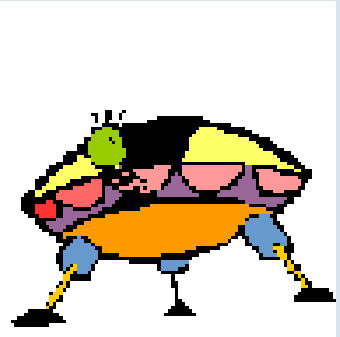
Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Reflecting on Standards for Mathematical Practice

Teacher Actions (Cause)	Student Practices (Effect)
1. Provide time for and facilitates the discussion of problem solution.	1. Make sense of problems and persevere in solving them. Do Students: • Use multiple representations (verbal descriptions, symbolic, tables, graphs, etc.)? • Explain to themselves the meaning of the problem. • Check their answers using different methods? • Continually ask, "Does this make sense?" • Understand the approaches of others and identify correspondences between different approaches?

There are 4 space ships. Each spaceship has 3 durdles (legs). There are 12 durdles all together.



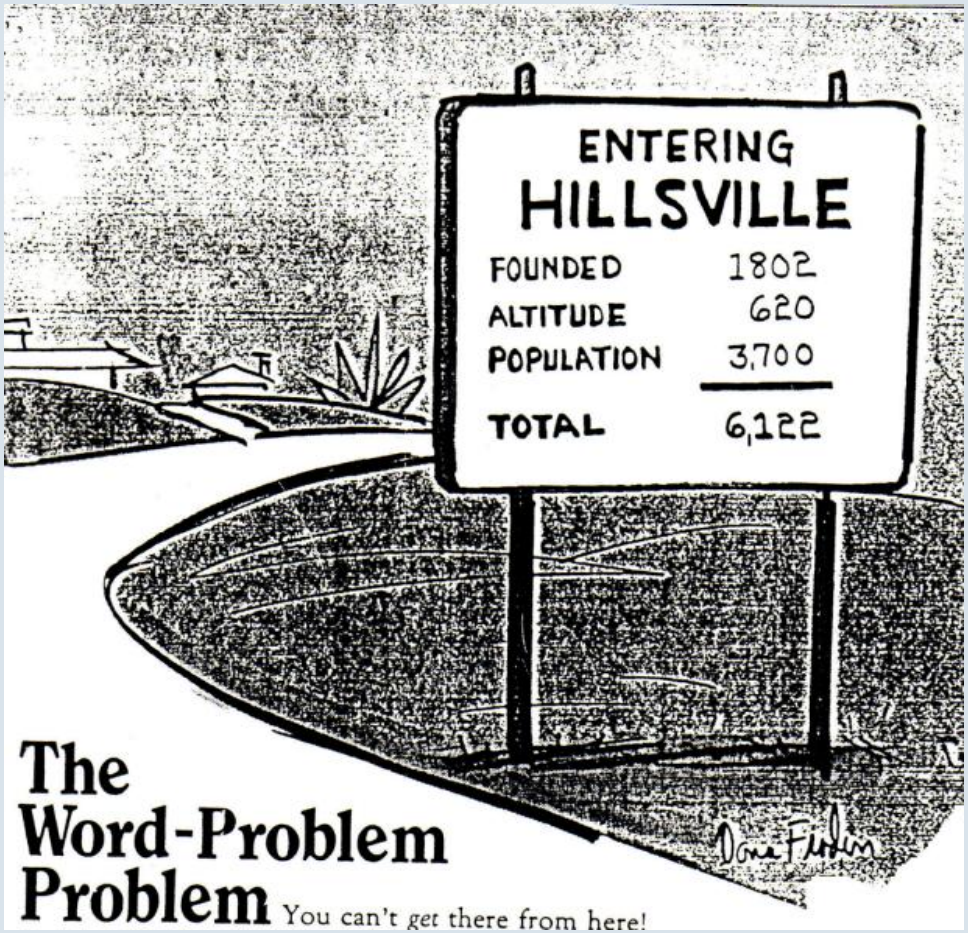
timer

1. Make sense of problems and persevere in solving them.

DO STUDENTS:

- **Use multiple representations (verbal descriptions, symbolic, tables, graphs, etc.)?**
- **Check their answers using different methods?**
- **Continually ask "Does this make sense?"**
- **Understand the approaches of others and identify correspondences between different approaches?**

1. Make sense of problems and persevere in solving them.

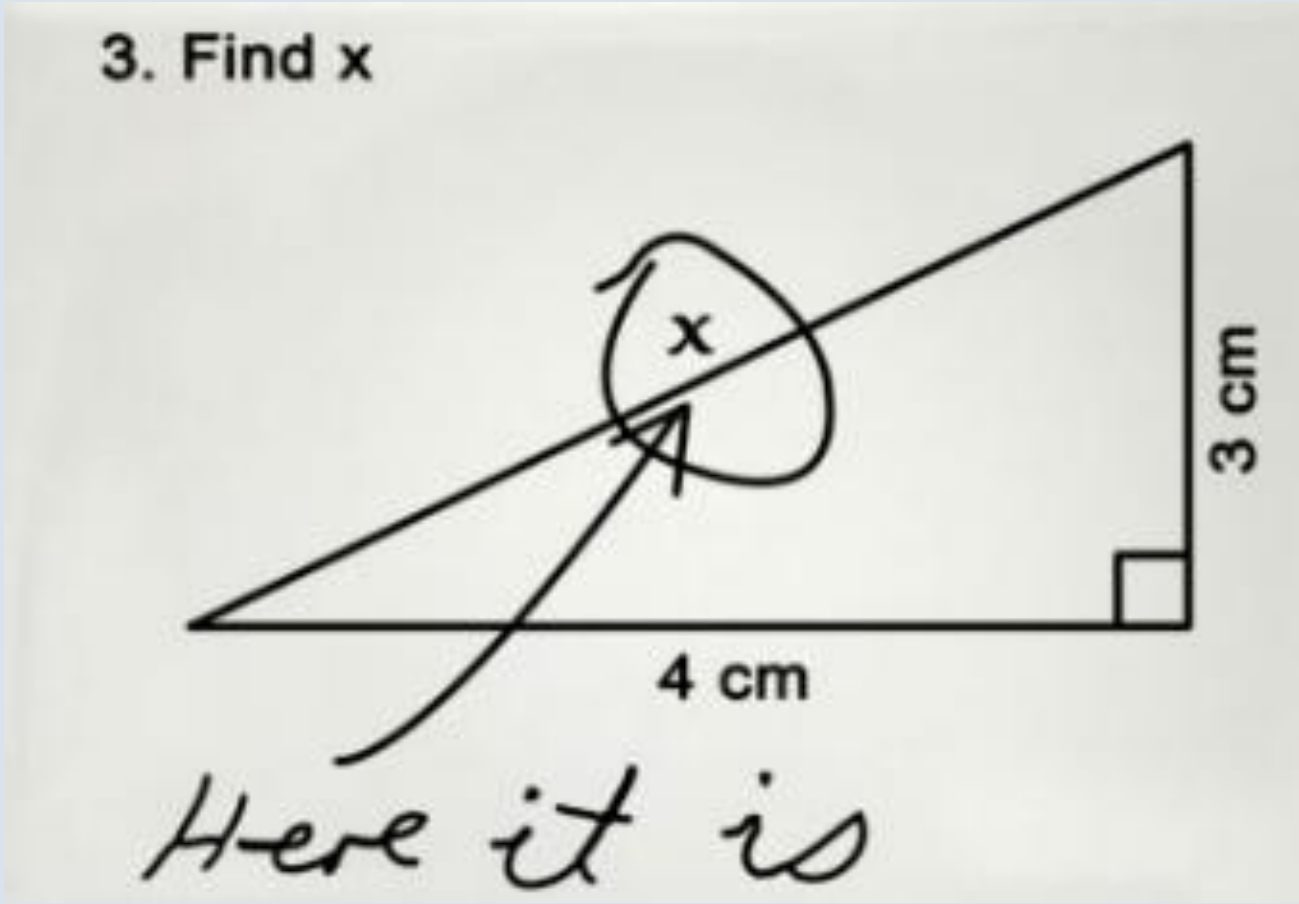


2. Reason abstractly and quantitatively.

DO STUDENTS:

- Make sense of quantities and their relationships in problem situations?
- Decontextualize a problem?
- Contextualize a problem?
- Create a coherent representation of the problem, consider the units involved, and attend to the meaning of quantities

2. Reason abstractly and quantitatively.



3. Construct viable arguments and critique the reasoning of others.

DO STUDENTS:

- Make conjectures and build a logical progression of statements to explore the truth of their conjectures?
- Analyze situations and recognize and use counter examples?
- Justify their conclusions, communicate them to others, and respond to arguments of others?
- Hear or read arguments of others and decide whether they make sense, and ask useful questions to clarify or improve the argument?

3. Construct viable arguments and critique the reasoning of others.

- 7.8 x .98
- 45.1 x 1.05
- 0.52 x 15.6

4. Model with mathematics.

DO STUDENTS:

- **Apply the mathematics they know to solve problems in everyday life?**
- **Apply what they know and make assumptions and approximations to simplify a complicated situation as an initial approach?**
- **Identify important quantities in a practical situation?**
- **Analyze relationships mathematically to draw conclusions?**
- **Interpret their mathematical results in the context of the situation and reflect on whether the results make sense?**

4. Model with mathematics.

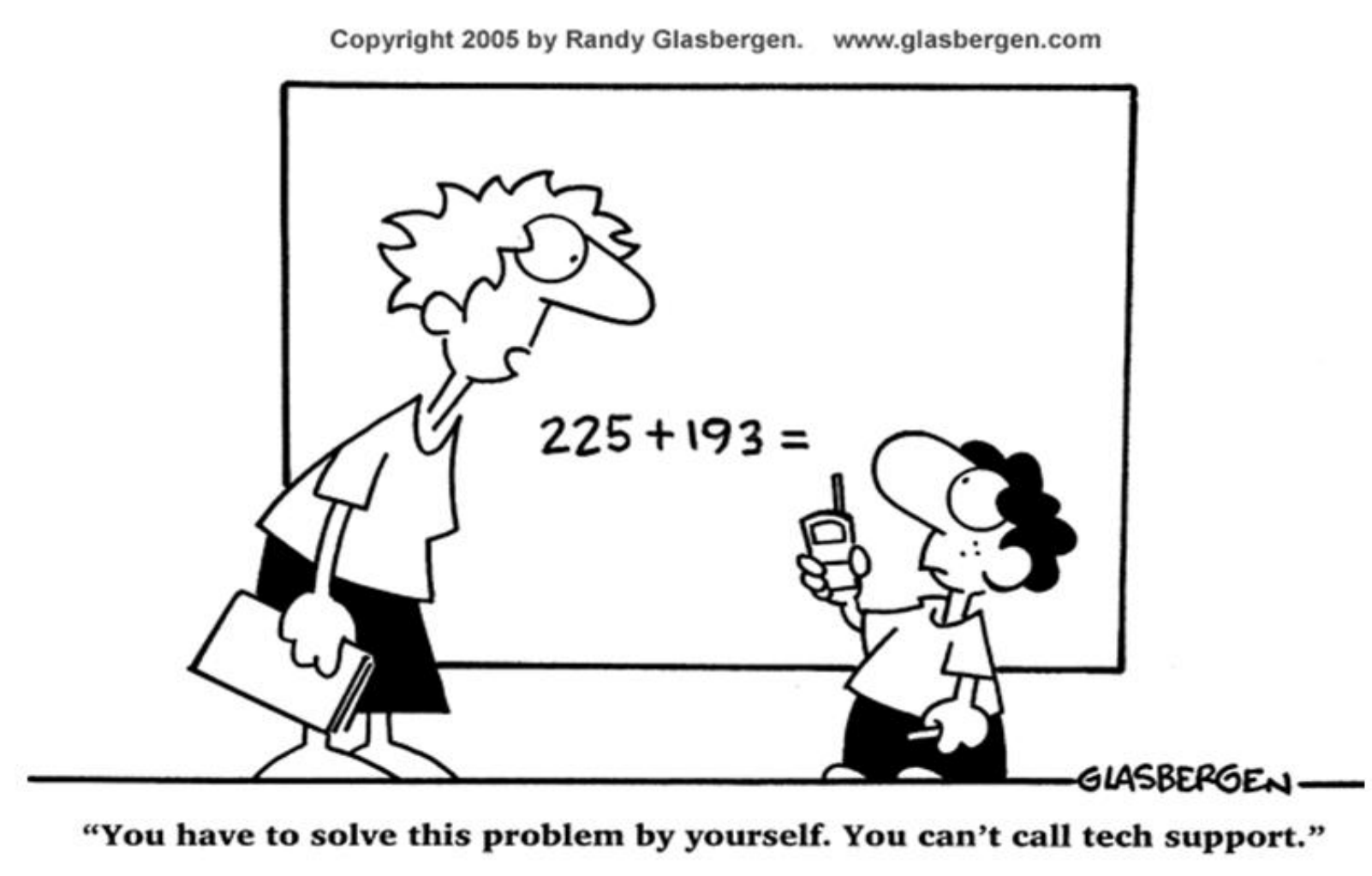
$$\frac{1}{2} + \frac{2}{3} =$$

5.Use appropriate tools strategically.

DO STUDENTS:

- Consider the available tools when solving mathematical problems?
- Know the tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful?
- Identify relevant external mathematical resources and use them to pose or solve problems?
- Use technological tools to explore and deepen their understanding of concepts?

5.Use appropriate tools strategically.

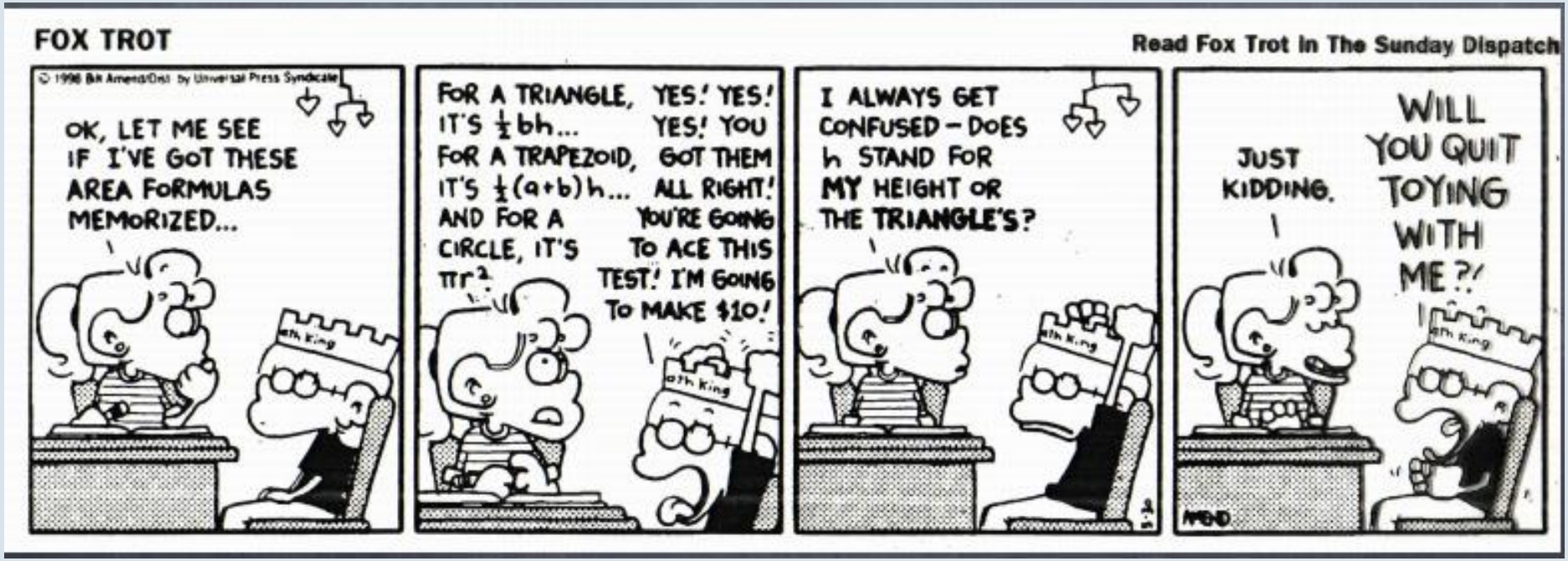


6. Attend to precision.

DO STUDENTS:

- Communicate precisely to others?
- Use clear definitions?
- Use the equal sign consistently and appropriately?
- Calculate accurately and efficiently?

6. Attend to precision.

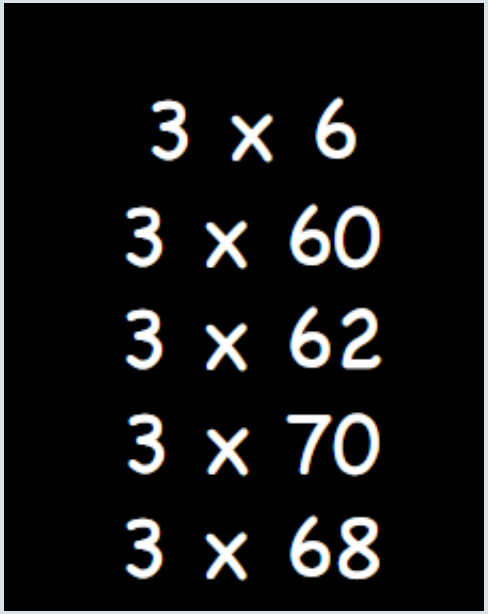


7. Look for and make use of structure.

DO STUDENTS:

- Look closely to determine a pattern or structure?
- Utilize properties?
- Decompose and recombine numbers and expressions?

7. Look for and make use of structure.



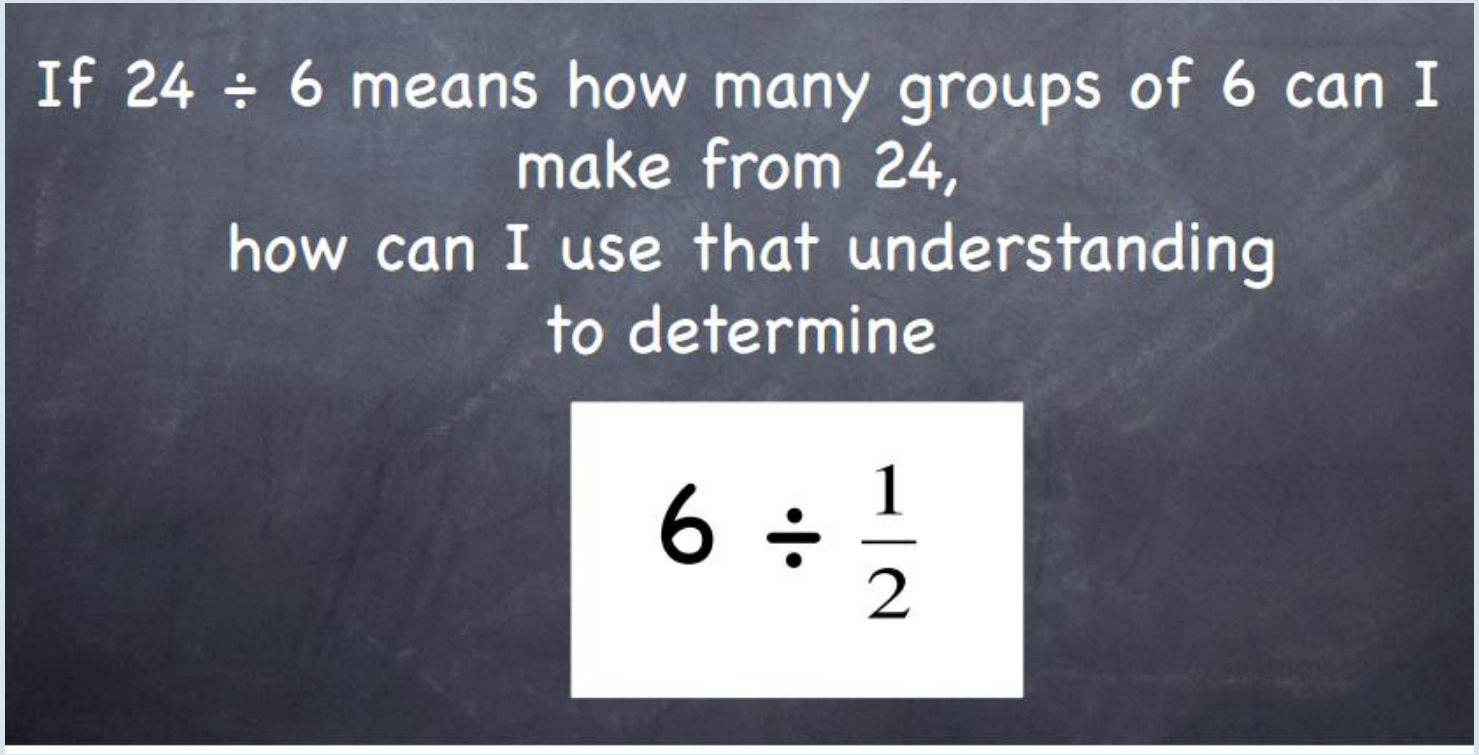
Where's the math?

8. Look for and express regularity in repeated reasoning.

DO STUDENTS:

- Notice if calculations are repeated, and look both for general methods and for shortcuts?
- Maintain oversight of the process, while attending to the details?
- Continually evaluate the reasonableness of their intermediate result?

8. Look for and express regularity in repeated reasoning.



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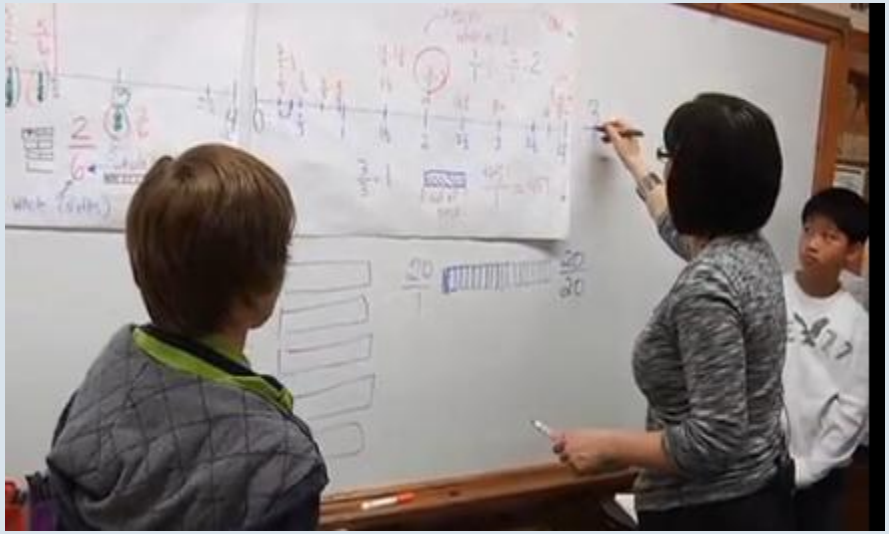
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Mathematical practices describe the habits of mind of mathematically proficient students…

- **Who is doing the talking?**
- **Who is doing the math?**

Students View Themselves as Mathematicians

Students as Mathematicians

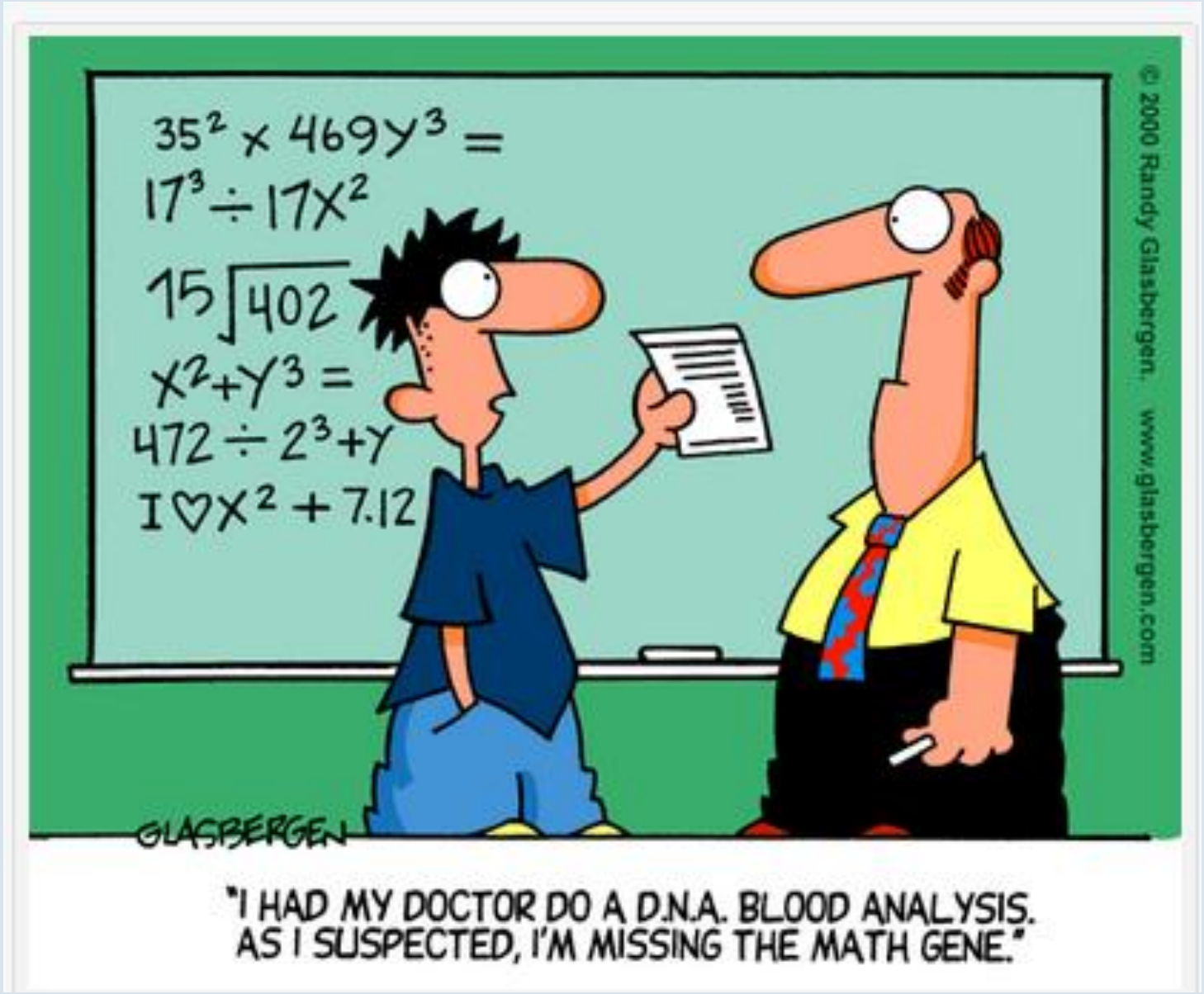


http://www.curriculum.org/secretariat/coplanning/

NC Teaching Standards

Time to Reflect

Summary	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$17^3 \div 17x^2$
$15\sqrt{402}$
$x^2+y^3 =$
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$I \heartsuit x^2 + 7.12$
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GLASBERGEN
"I HAD MY DOCTOR DO A D.N.A. BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."

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5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Composing and Decomposing Number

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

c. Understand that each successive number name refers to a quantity that is one larger.

K.CC.5 Count to answer "how many?" questions about **as many as 20 things** arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

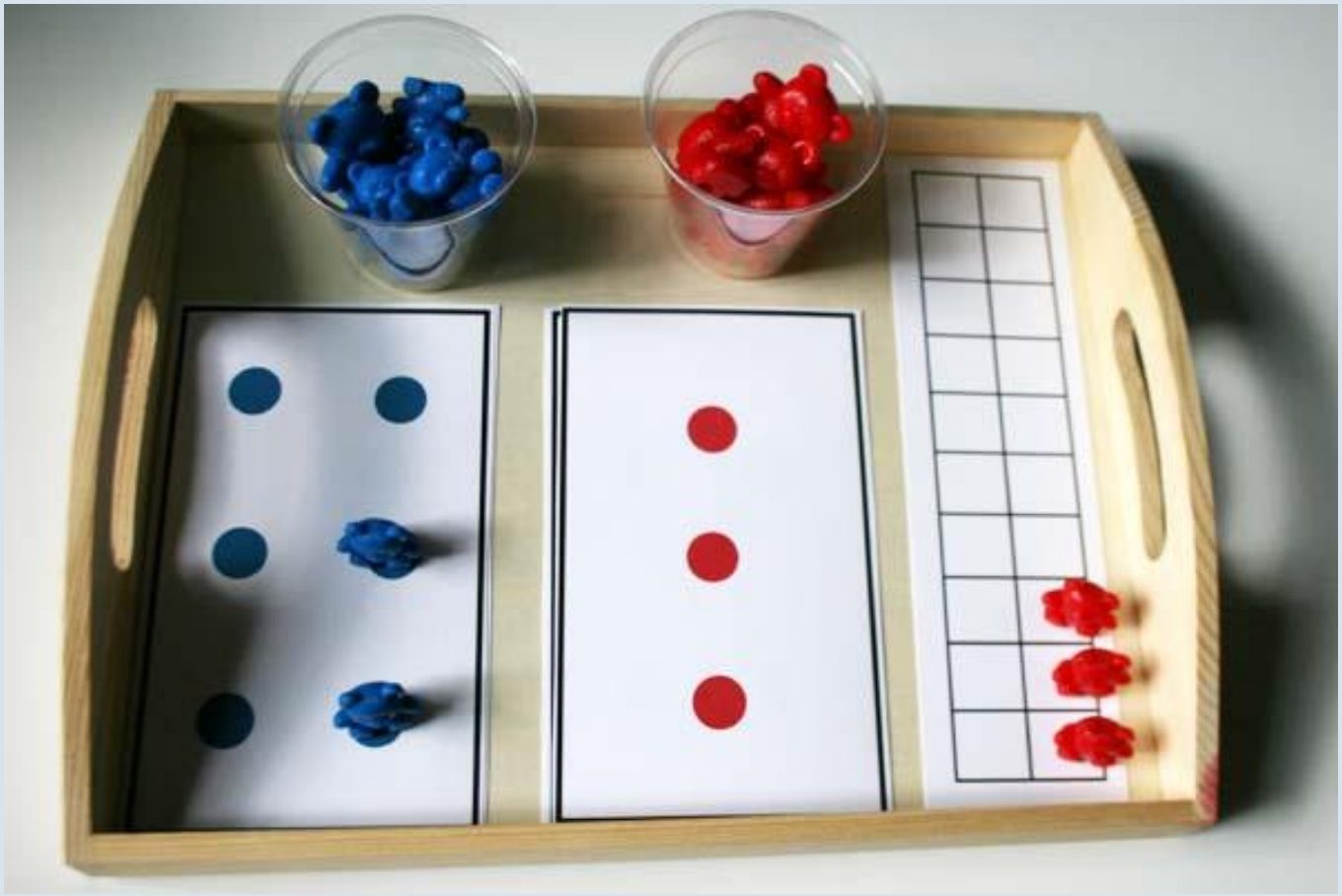
The purpose of counting

is

to answer the question,

“How Many?”

One-to-One Correspondence



One-to-One Tagging

Cardinality

Keeping Track

Inclusion

Subitize

Turn and Talk

What different contexts could be provided to support the development of counting at home and at school?

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

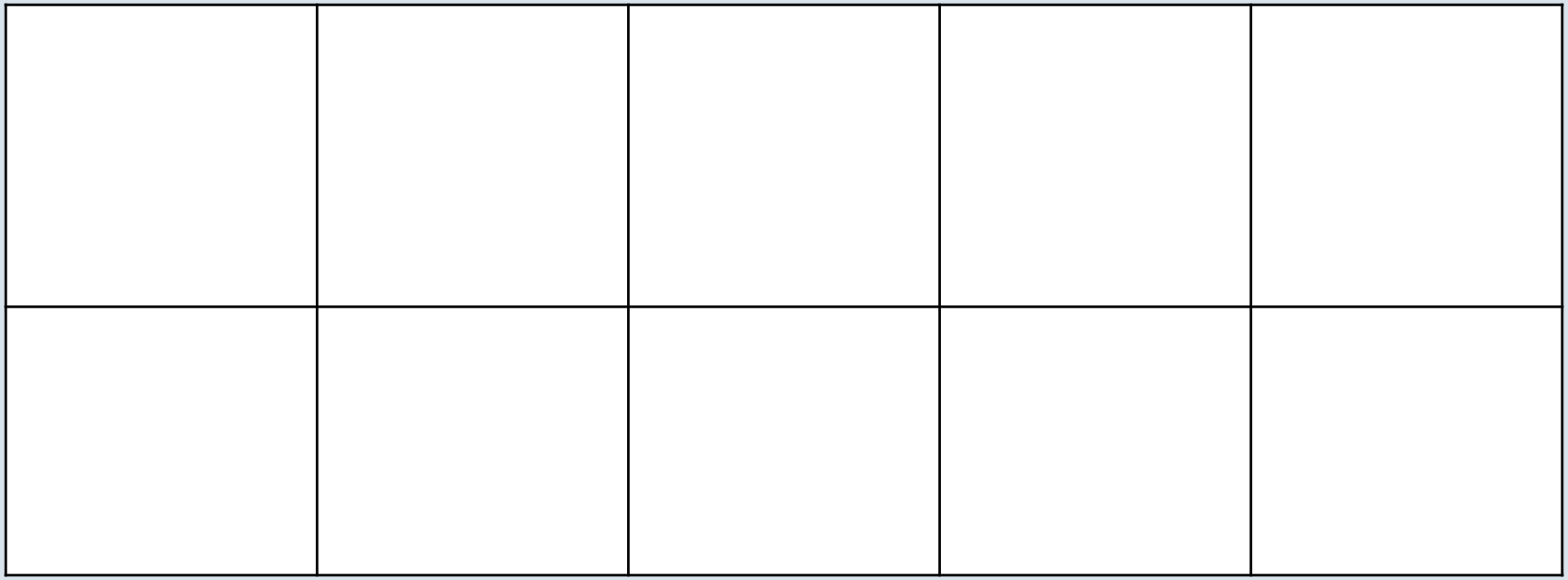
K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

K.OA.4 For any number from 1 to 9, **find the number that makes 10** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

K.0A.5 Fluently add and subtract **within 5**.

Let's Make Tens

- Get a partner, some color tiles, and a ten frame.



Let's Make Tens

- Partner 1 draws a number card and fills in the ten frame.
- Partner 2 tells how many are needed to make ten.
- Clear the frame.
- Now the other partner draws a number and fills the frame.

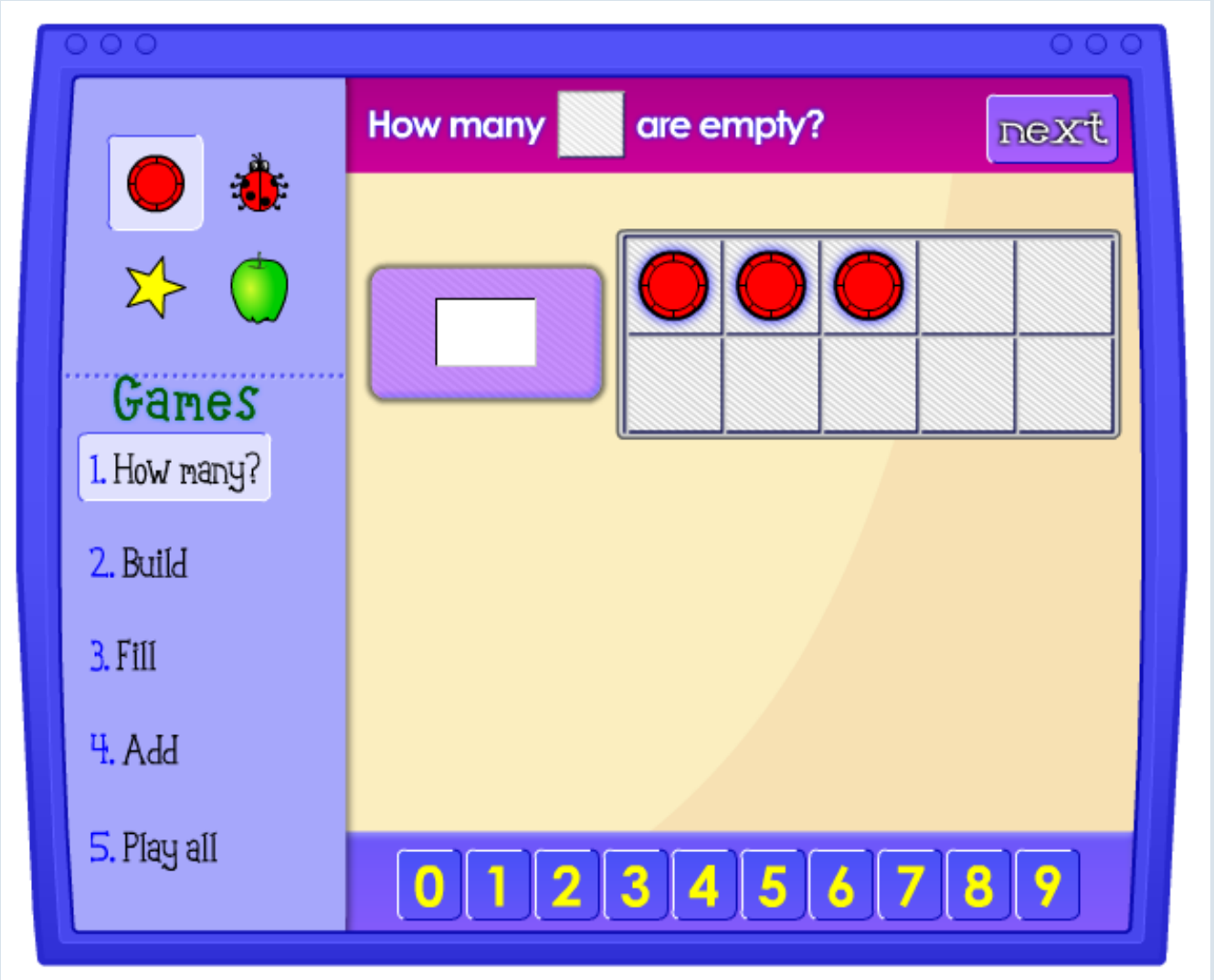
Turn and Talk

What did you notice…

- **about ways to make ten?**
- **about learning basic facts?**
- **about the importance of visual and kinesthetic representation?**

Turn and Talk

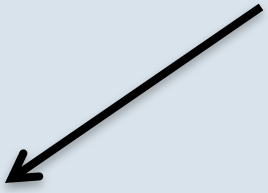
- Does this task have multiple entry points?
- How can it be differentiated?
- Will it help ELL students build understanding?



http://illuminations.nctm.org/ActivityDetail.aspx?ID=75

Research

Students are shown this number. Teacher points to the 6 and says, "Can you show me this many?"



16

Research

When the teacher points to the 1 in the tens place and asks, "Can you show me this many?"

16

Research

By third grade nearly half the students still do not 'get' this concept.

16

More research - It gets worse!

**A number contains 18 tens,
2 hundreds, and 4 ones.
What is that number?**

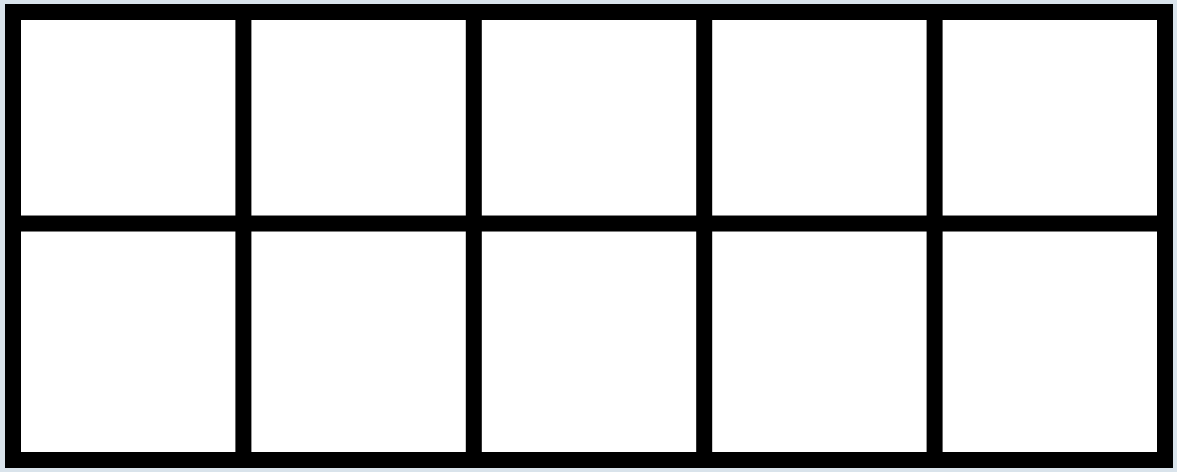
1824 **2824**

218.4 **384**

Common Core State Standards begin to specifically address this misunderstanding in Kindergarten and First Grade.

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.



Understand place value.

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Understand place value.

1.NBT.3 Compare 2 two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.5 Given a two-digit number, **mentally find 10 more or 10 less** than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Ten Frames for Addition and Subtraction

Using the filled ten frames and the partially filled ones, create the number 45. Now subtract 20.

Brainstorm ways to use the ten frames to create tasks for first graders.

Turn and Talk

How does this cluster build understanding of place value?

How is this different from the way we have traditionally taught place value?

35 + 23

Using decomposing of number and the associative property, second graders may decide that since

35 = 30 + 5 and 23 = 20 + 3,

they can add 20 + 30 = 50 and 5 + 3 = 8, then add 50 + 8 = 58

35 + 23

Decompose only the 23

35 + 20 = 55

then

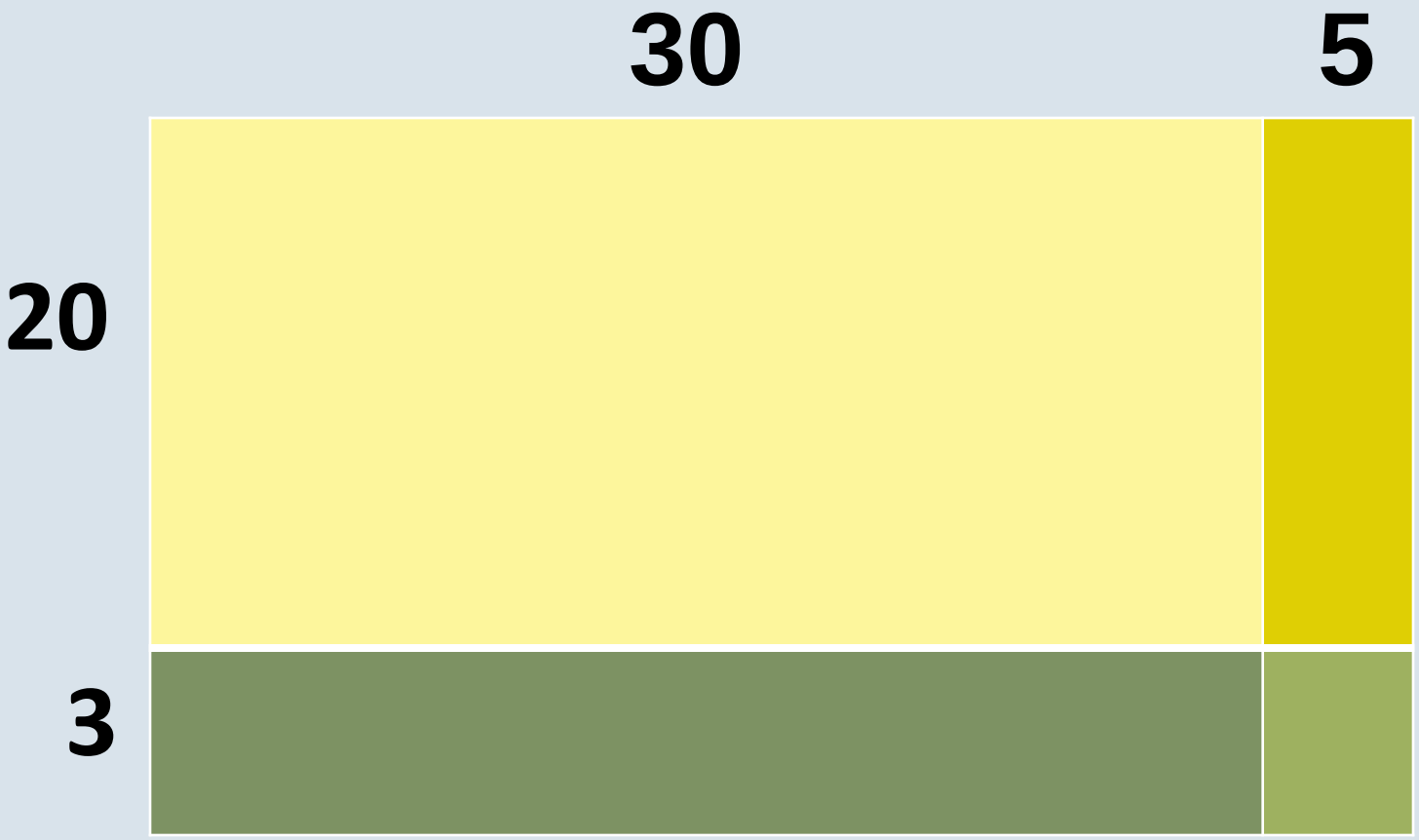
55 + 3 = 58

The distributive property can and should be used to teach multiplication.

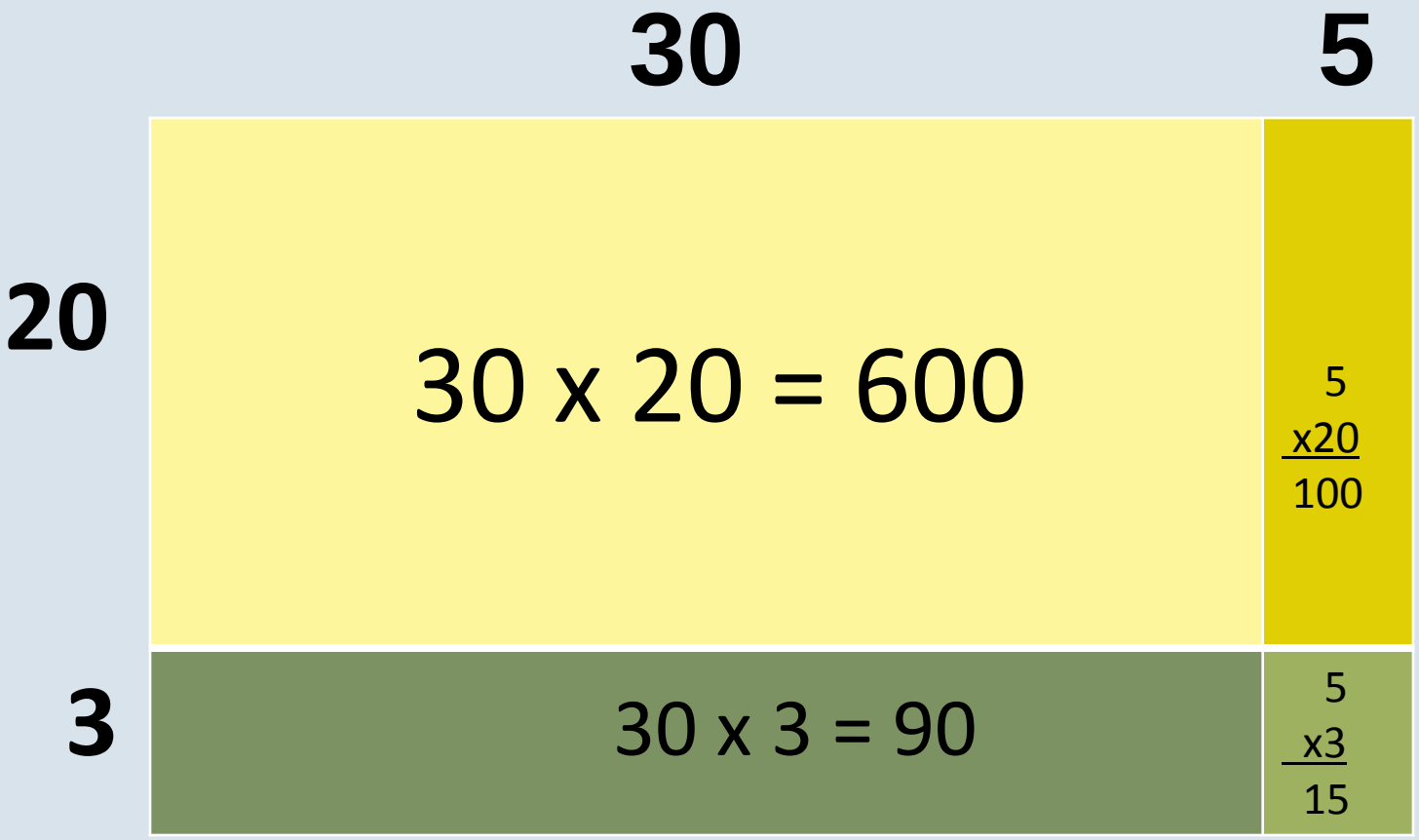
35 X 23

Let's use an area model.

35 x 23



35 x 23



600 + 100 + 90 + 15 = **805**

$$
\begin{array}{r}
35 \\
\times\ 23 \\
\hline
15 \\
90 \\
100 \\
600 \\
\hline
805
\end{array}
$$

“With each extension of number, the meanings of addition, subtraction, multiplication and division are extended.”

Common Core State Standards

“In each new number system- integers, rational numbers, real numbers, and complex numbers - the four operations remain the same in two important ways: they follow the same properties and their meanings are consistent with their previous meanings.”

Common Core State Standards

“Arithmetic is a rehearsal for algebra.”

Bill McCullam, CCSS Mathematics Author,

Time to Reflect

Summary	

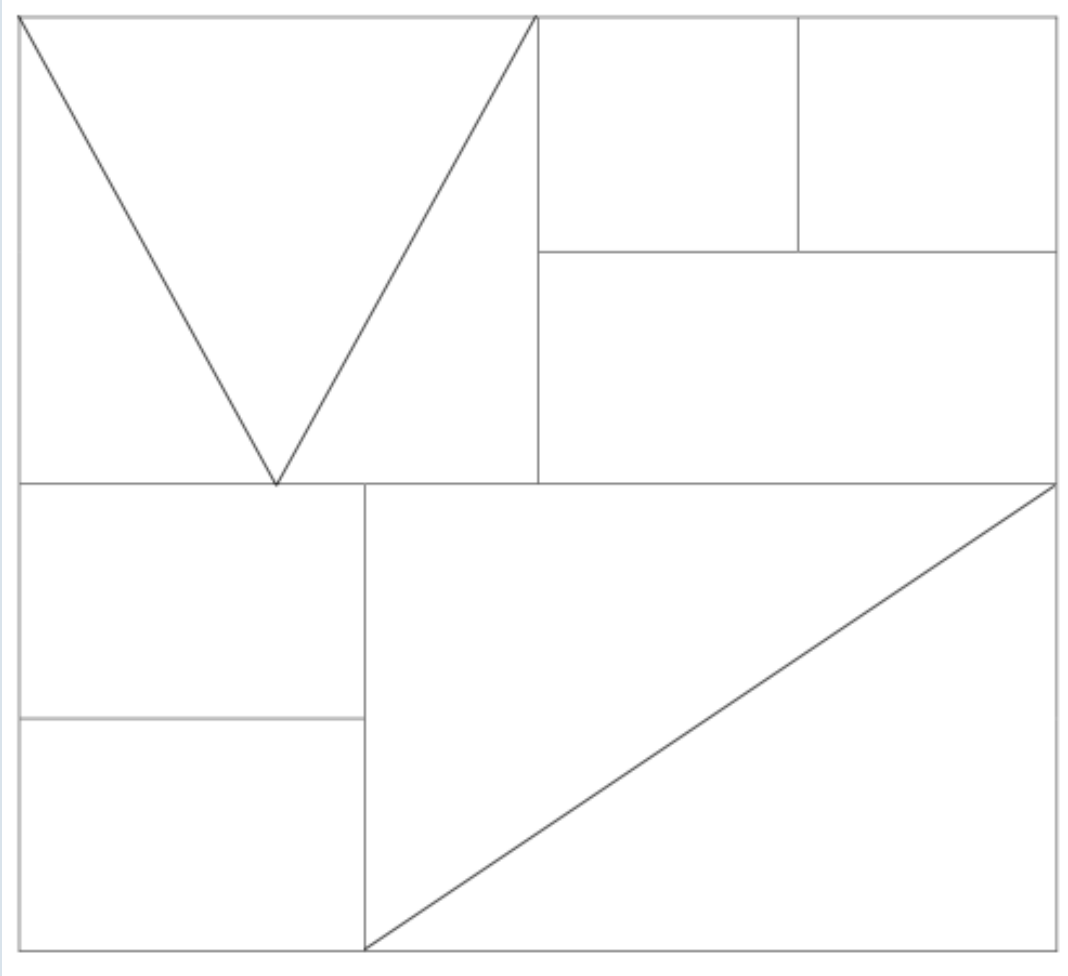

© 2001 Randy Glasbergen. www.glasbergen.com
$80,000 + 0 = $800,000
Zero pay raise for the CEO who is bad at Math
GLASBERGEN
"That's right, I've decided to give myself zero pay raise this year."

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Fractions

If the square = 1 whole, what is the value of each piece?



Fractions are a rich part of mathematics, but we tend to **manipulate fractions by rote rather than try to make sense of the concepts and procedures**.

Fractions are a rich part of mathematics, but we tend to **manipulate fractions by rote rather than try to make sense of the concepts and procedures**. Researchers have conclude that this complex topic causes **more trouble for students than any other area of mathematics**.

Bezuk and Bieck 1993

Turn and Talk

Where are fractions in the K-2 standards and how are they represented?

timer

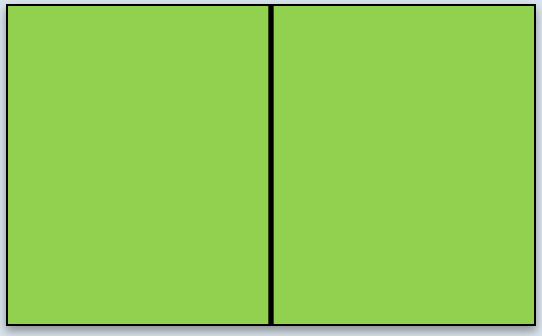
Reason with shapes and their attributes.

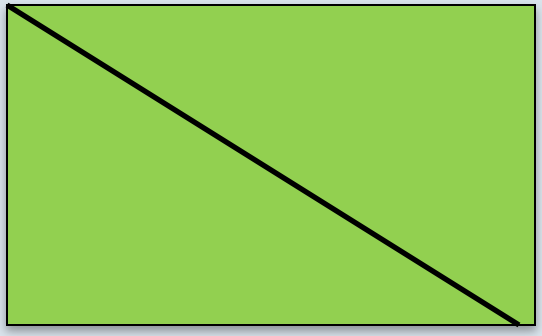
1.G.3 Partition circles and rectangles into two and four **equal shares**, describe the shares using the words ***halves*****,** ***fourths*****, and *quarters*,** and use the phrases ***half of*****,** ***fourth of***, and ***quarter of*****.** Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates **smaller shares.**

Reason with shapes and their attributes.

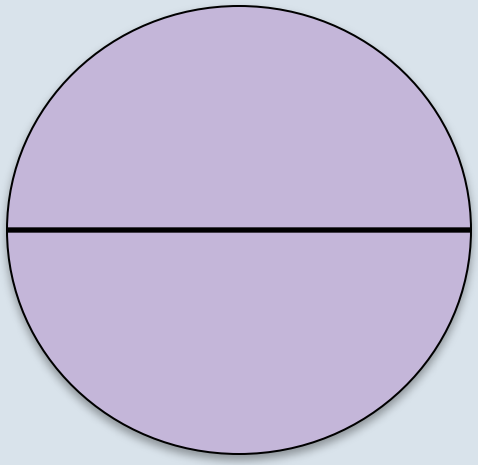
1.G.3 Partition circles and rectangles into two and four **equal shares**, describe the shares using the words ***halves*****, *fourths*****, and *quarters*****,** and use the phrases ***half of*****, *fourth of***, and ***quarter of*****.** Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

How can you and a friend share equally (partition) a piece of paper so that you both have the same amount of paper?



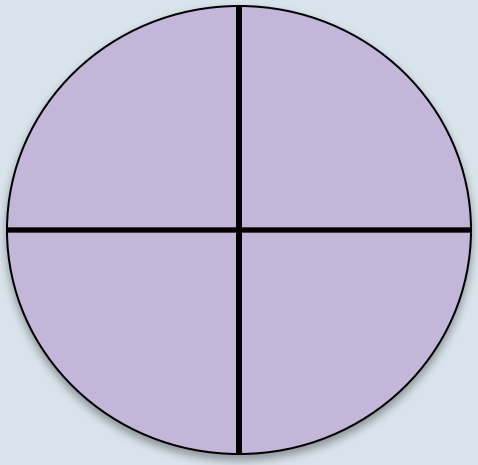


There is a pizza for dinner. What do you notice about the slices on the pizza?



"There are two slices on the pizza. Each slice is the same size. Those are big slices!"

If we cut the same pizza into four slices (fourths), do you think the slices would be the same size, larger, or smaller?



"When you cut the pizza into fourths. The slices are smaller. More slices mean that the slices get smaller and smaller."

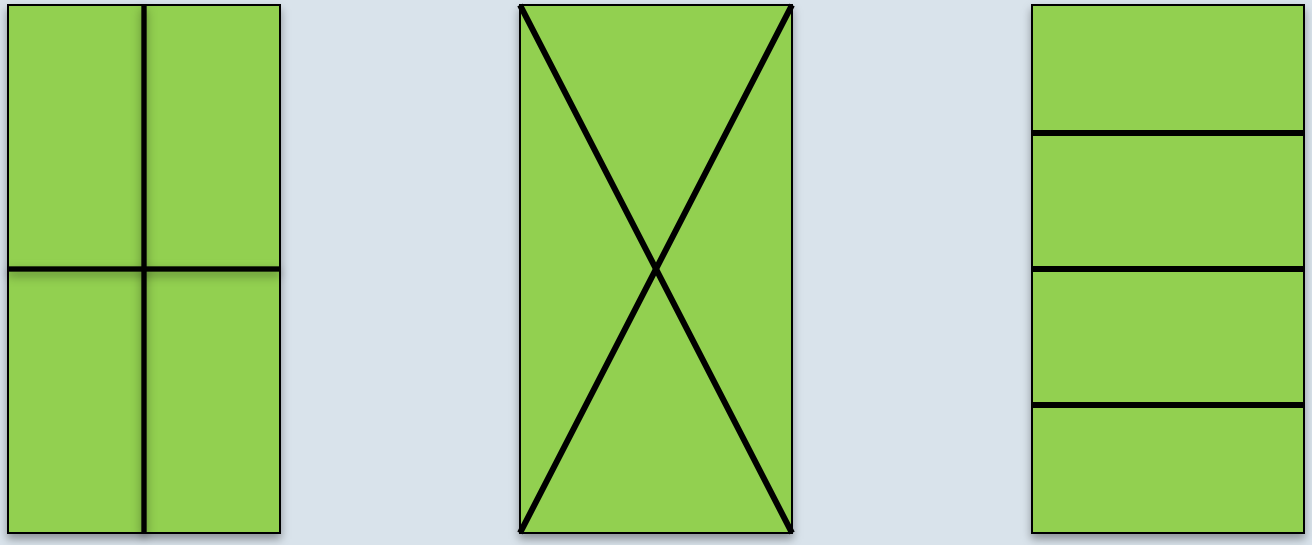
Reason with shapes and their attributes.

2.G.3 Partition circles and **rectangles** into **two, three, or four equal shares**, describe the shares using the words **halves, thirds, half of, a third of**, etc., and describe the whole as **two halves, three thirds, four fourths.** Recognize that **equal shares** of identical wholes need not have the same shape.

Reason with shapes and their attributes.

2.G.3 Partition circles and **rectangles** into **two, three, or four equal shares**, describe the shares using the words **halves, thirds, half of, a third of**, etc., and describe the whole as **two halves, three thirds, four fourths.** Recognize that **equal shares** of identical wholes need not have the same shape.

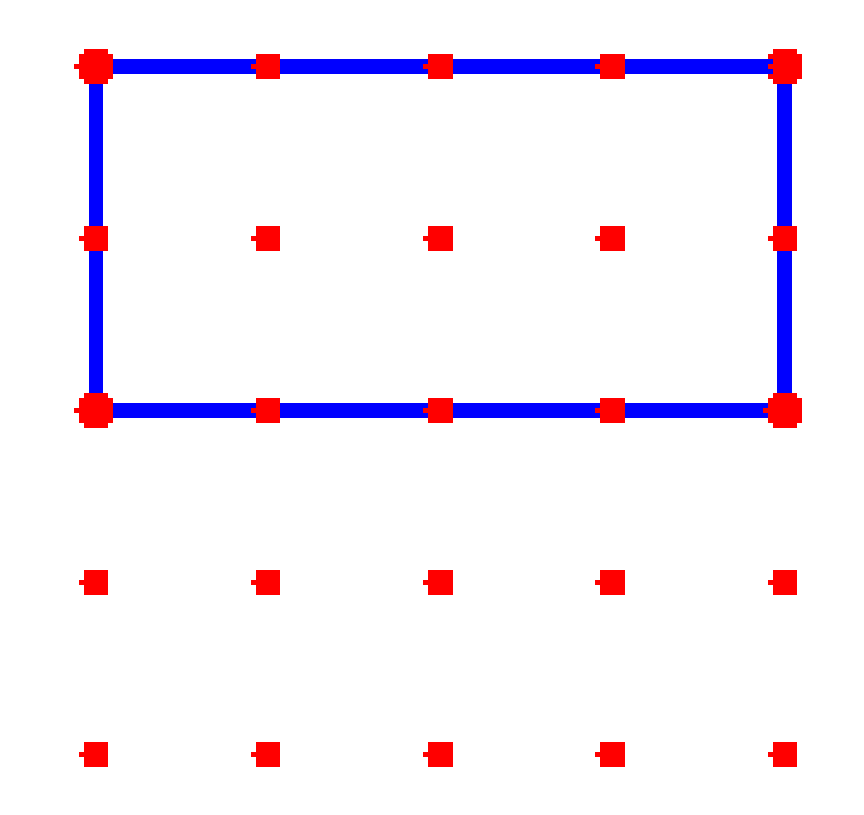
Divide (partition) each rectangle into fourths a different way.



Now Let's Do Some Math!

Geoboard Fractions

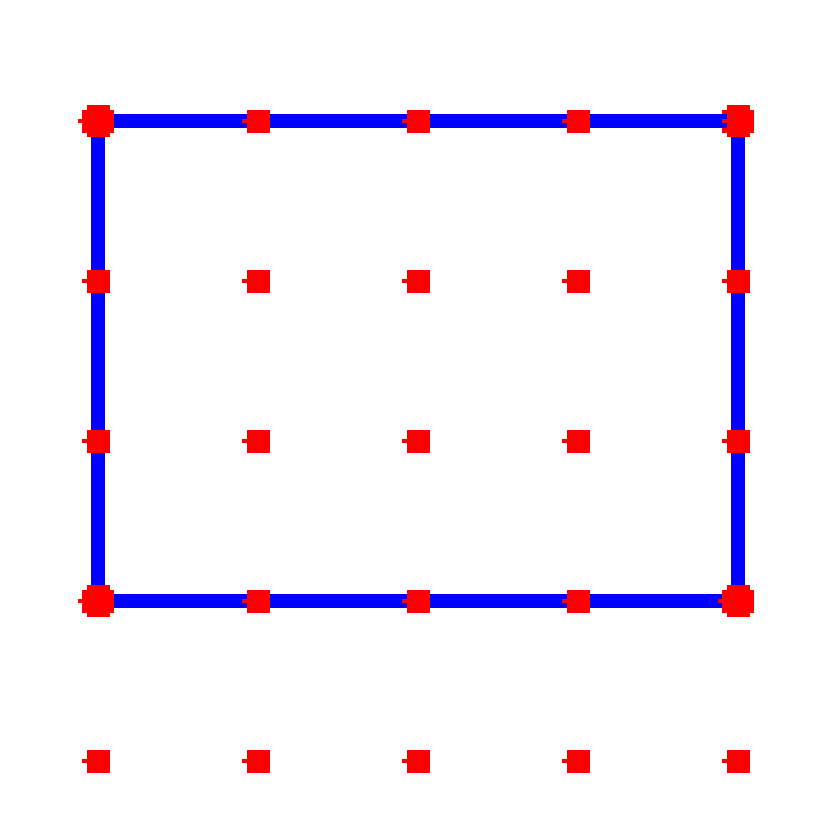
Make this rectangle on your geoboard.



Find ways to divide the rectangle in halves.

Geoboard Fractions

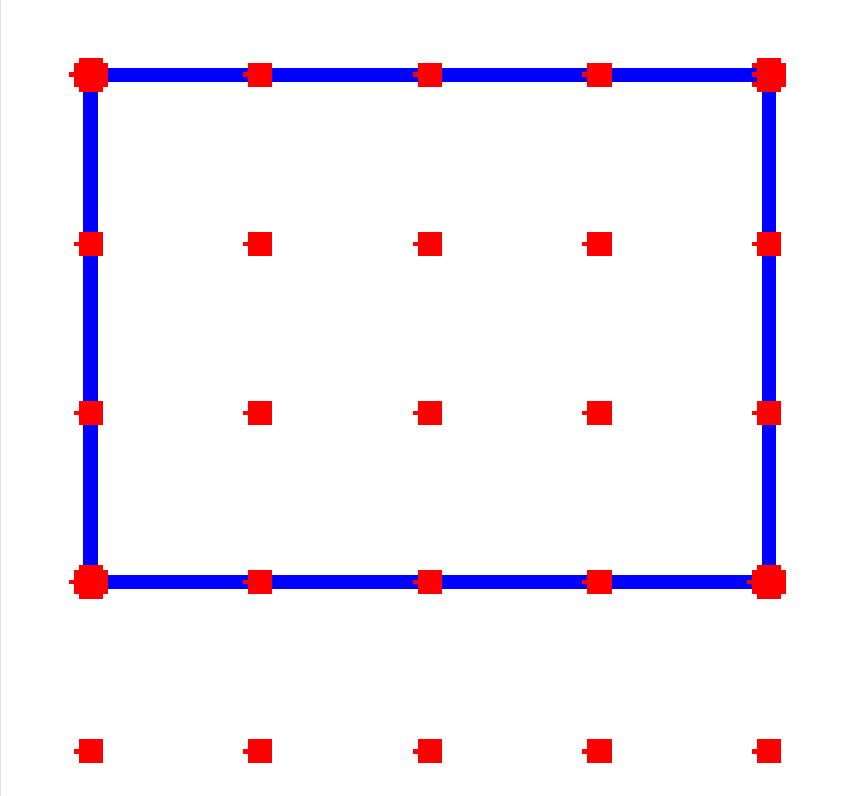
Make this rectangle on your geoboard.



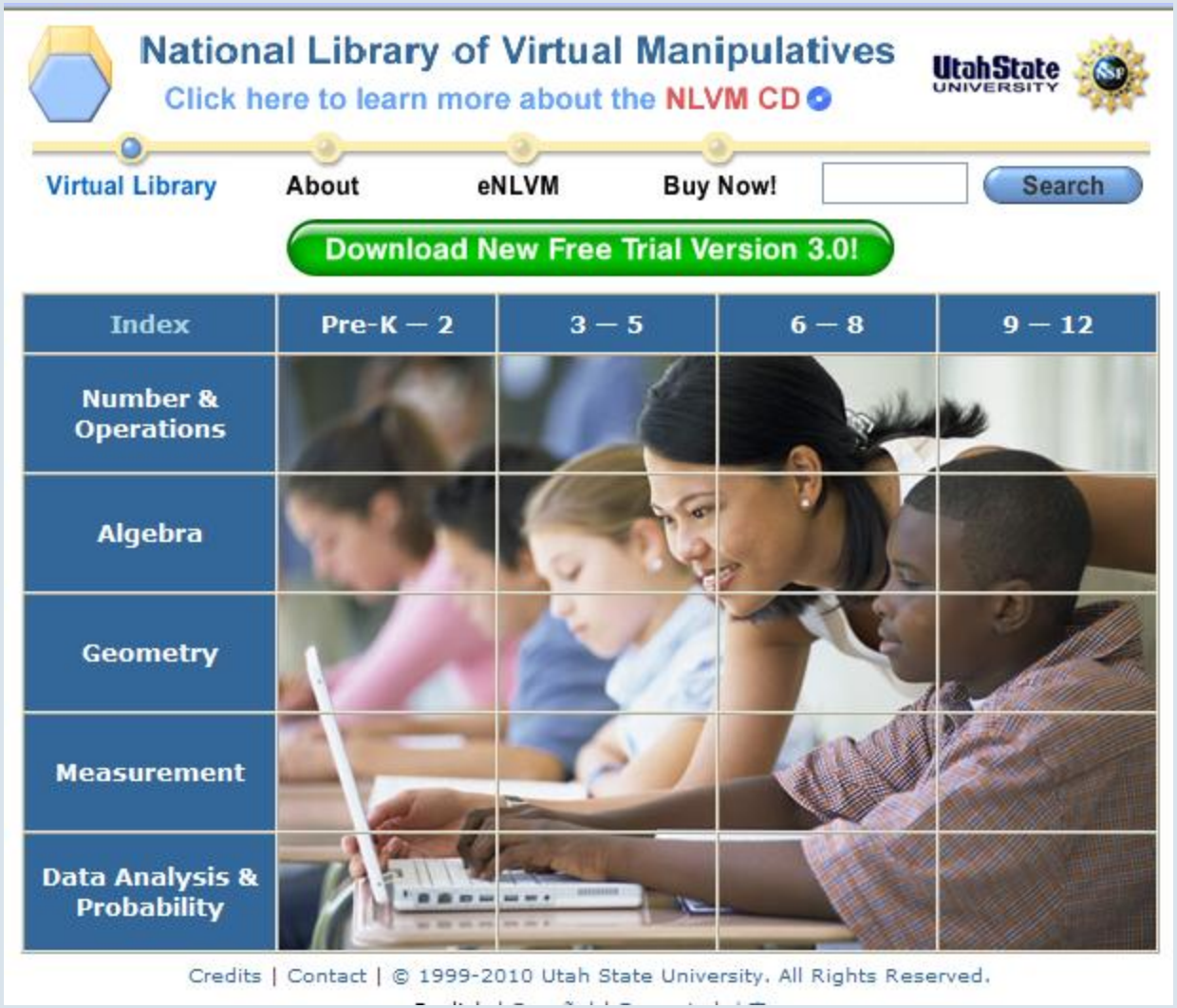
Find ways to divide the rectangle in thirds.

Geoboard Fractions

Make this rectangle on your geoboard.



Find ways to divide the rectangle in fourths.

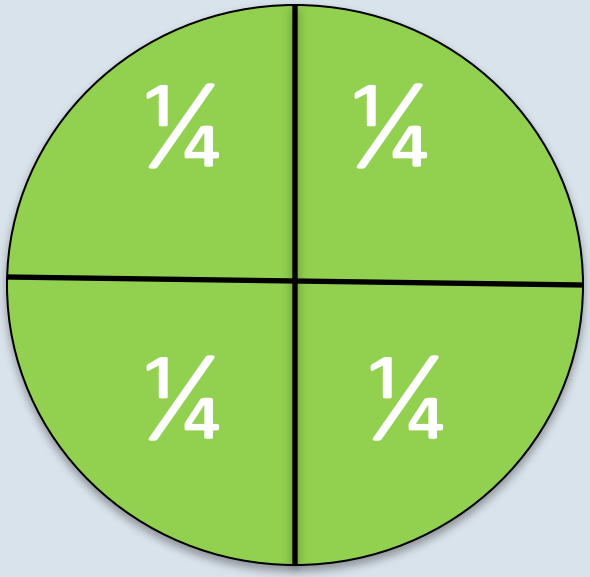


http://nlvm.usu.edu/en/nav/vlibrary.html

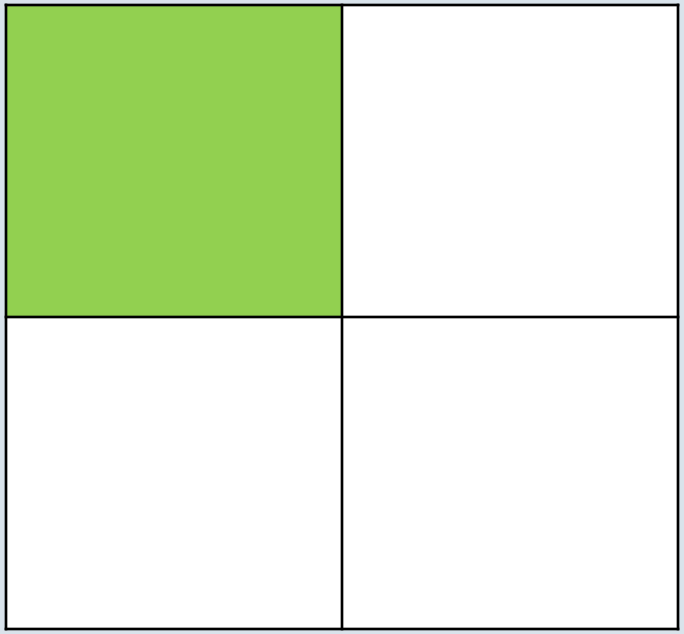
Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with **equal areas**. Express the area of each part as a **unit fraction** of the whole.

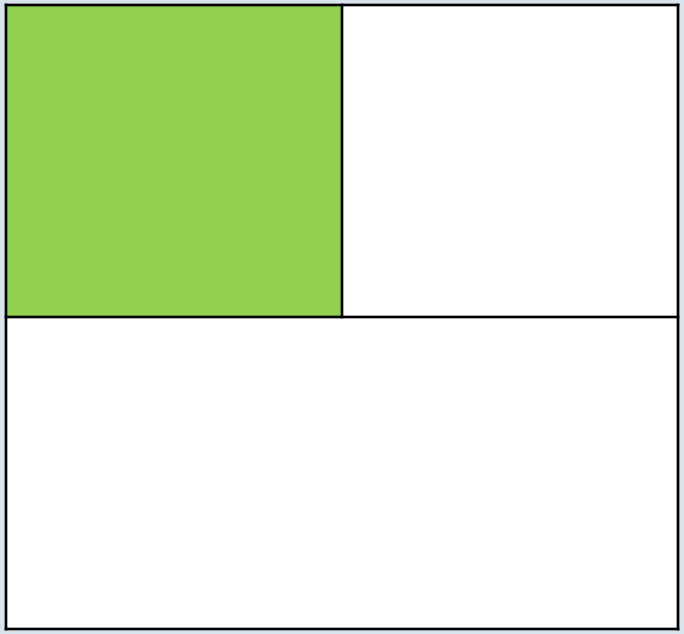
For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.



Write a fraction to show how much of the large square is shaded.



Write a fraction to show how much of the large square is shaded.



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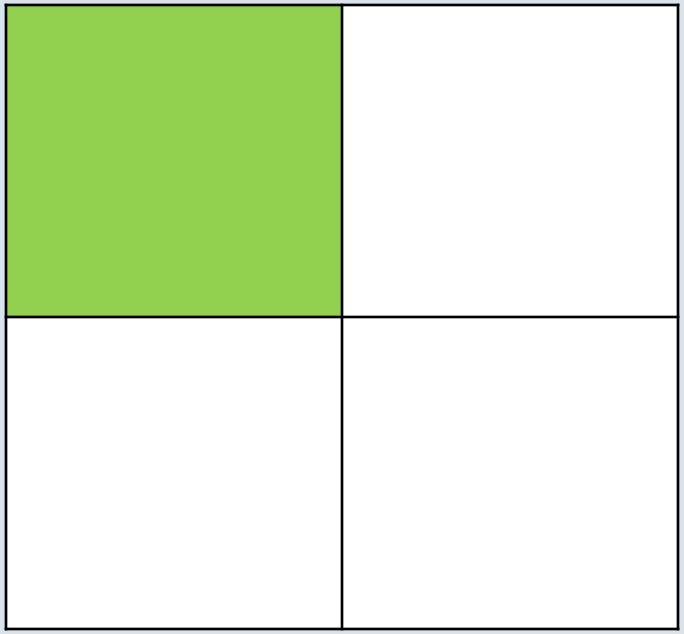


$$1 \div 4$$

First Grade

1.G.3 Partition circles and r**ectangles** into **two** and **four** equal shares, describe the shares using the words ***halves*****,** ***fourths*****,** and ***quarters*****,** and use the phrases ***half of*****,** ***fourth of*****,** and ***quarter of*****.** Describe the whole as **two of,** or **four of** the shares. Understand for these examples that **decomposing** into more equal shares creates smaller shares.

Second Grade

2.G.3 Partition circles and **rectangles** into **two, three**, or **four** equal shares, describe the shares using the words **halves, thirds, half of, a third of**, etc., and describe the whole as **two halves, three thirds, four fourths.** Recognize that equal shares of identical wholes need not have the same shape.

Third Grade

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a **unit fraction** of the whole.

Part-whole Relationship Research

- Partitioning wholes into equal-size pieces
- Identifying different units

This is the best way to approach learning about fractions in the early grades. It is essential for students to be provided opportunity to reason about the meaning of part-whole relations.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in **a larger unit in terms of a smaller unit**. Record measurement equivalents in a two column table.

For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

Symbolic representation of fractions

Instead of viewing a number such a 34 as representing a **specific quantity**, when the same digits (3 and 4) are used in the number ¾, the digits represent a **relationship**.

Develop understanding of fractions as numbers.

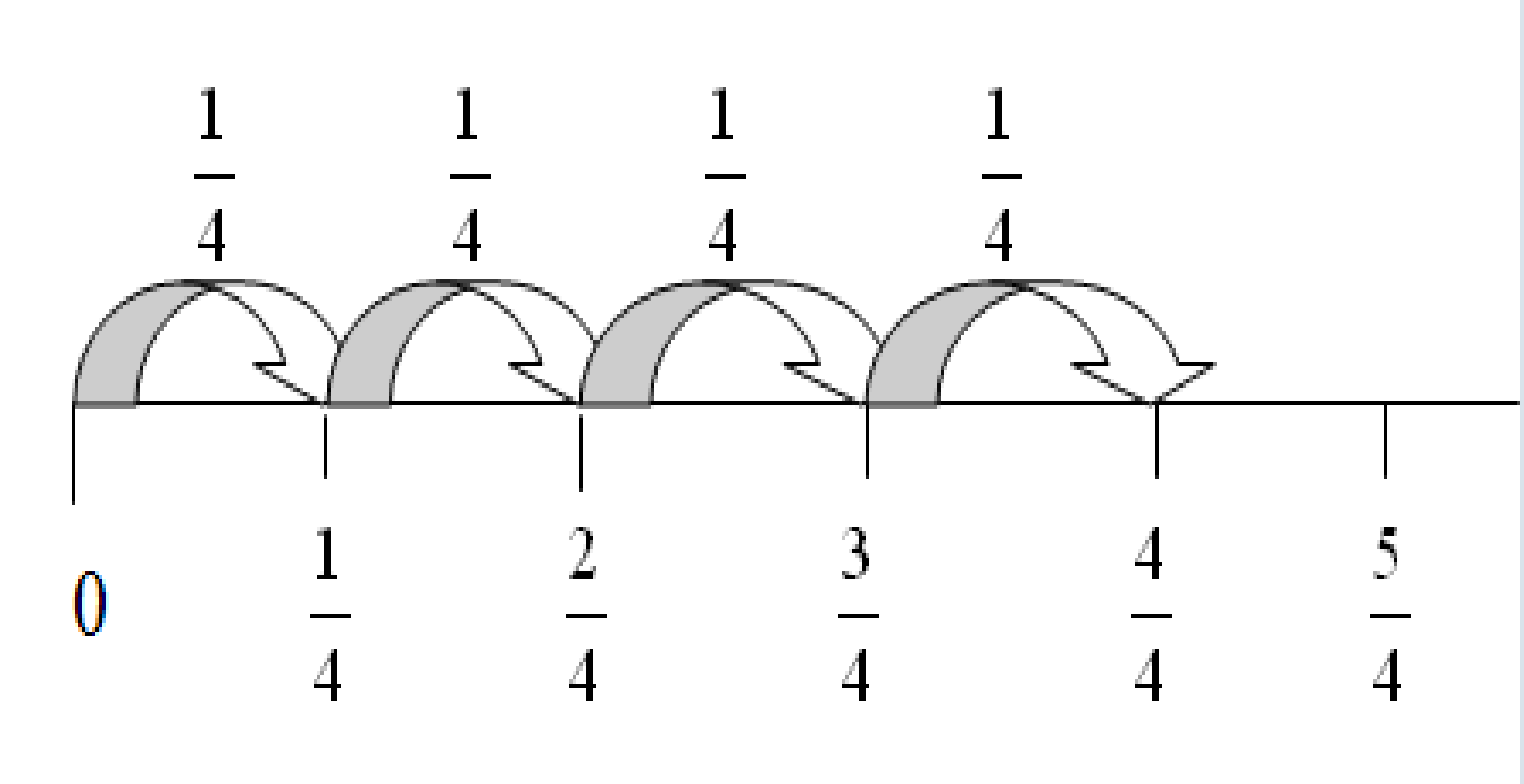
3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

3.NF.2 Understand a fraction as a number on the **number line**; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

– [5]Grade 3 expectations in this domain are limited to fraction with denominators 2, 3, 4, 6, and 8


1/4
1/4
1/4
1/4
0
1/4
2/4
3/4
4/4
5/4

What happens to the value of the fraction if….

- Numerator is increased by 1?
- Denominator is decreased by 1?
- Denominator is increased by 1?

Understanding

- Avoids too much emphasis on procedure
- Facilitates flexibility
- Justified conclusions
- Application to practical situations
- Accurate and clear expectations
- Metacognition

Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on **a number line.**

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

– **[5]Grade 3 expectations in this domain are limited to fraction with denominators 2, 3, 4, 6, and 8**

Turn and Talk
Third Grade Fractions

- What math tools could you use to teach third grade fractions?
- What are teachers currently using?
- What will be a challenge for them when teaching fractions in third grade?
- How will we help them overcome these challenges?

timer

Number Line

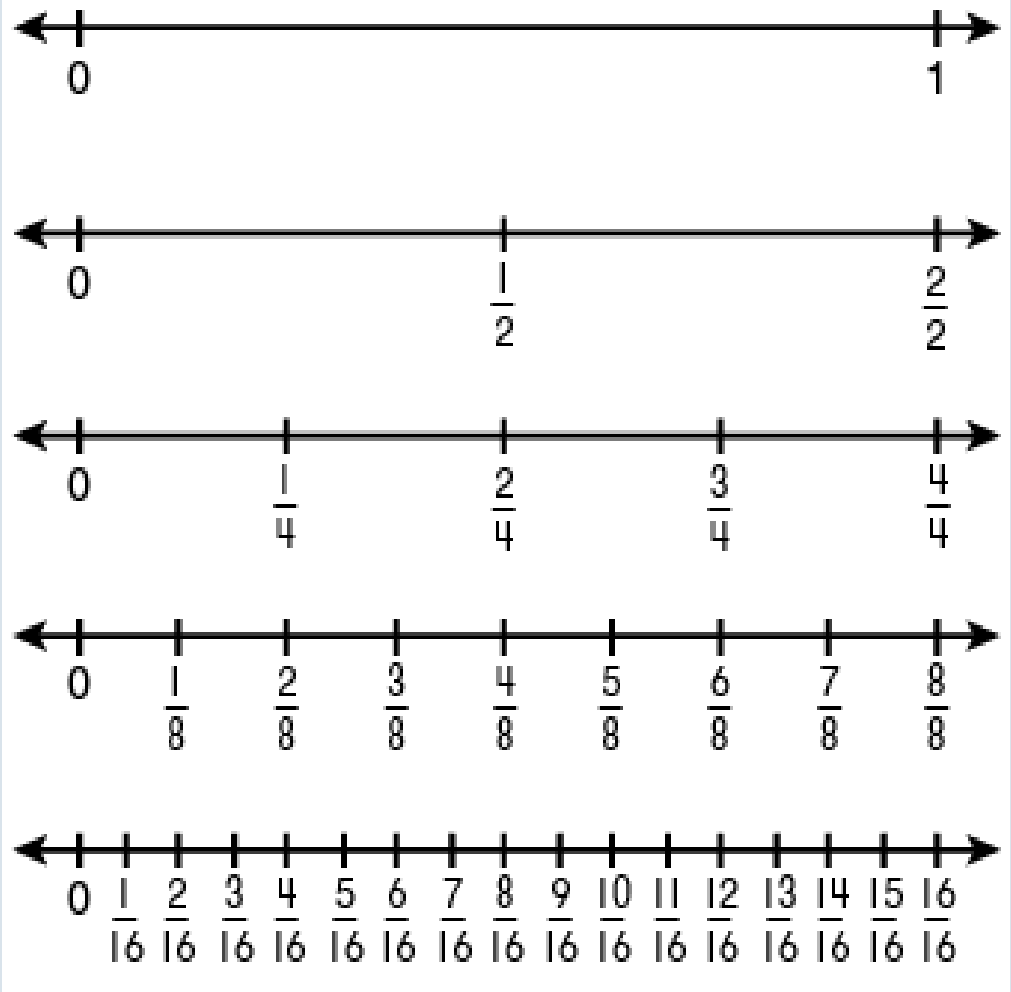
Featured prominently in the Common Core Content Standards as a model for representing numbers

Not only does use of the number lines persist across grade levels, but also across domains.

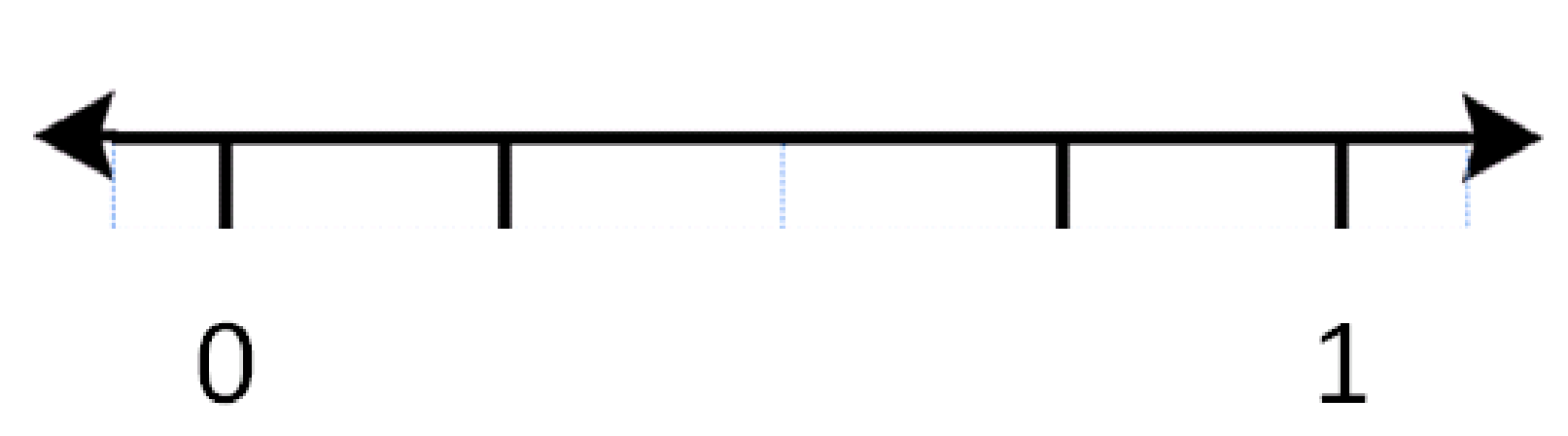
Number Line

- At your table is an envelope with information about the number line.
- Each person at the table is to take a slip of paper from the envelope, read it, and reflect.
- Then share information to others at your table.

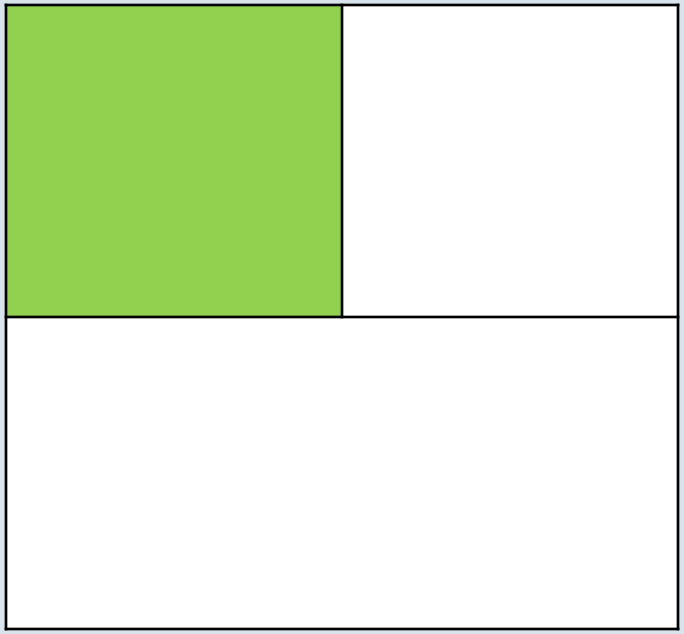
Fractions on a Number Line



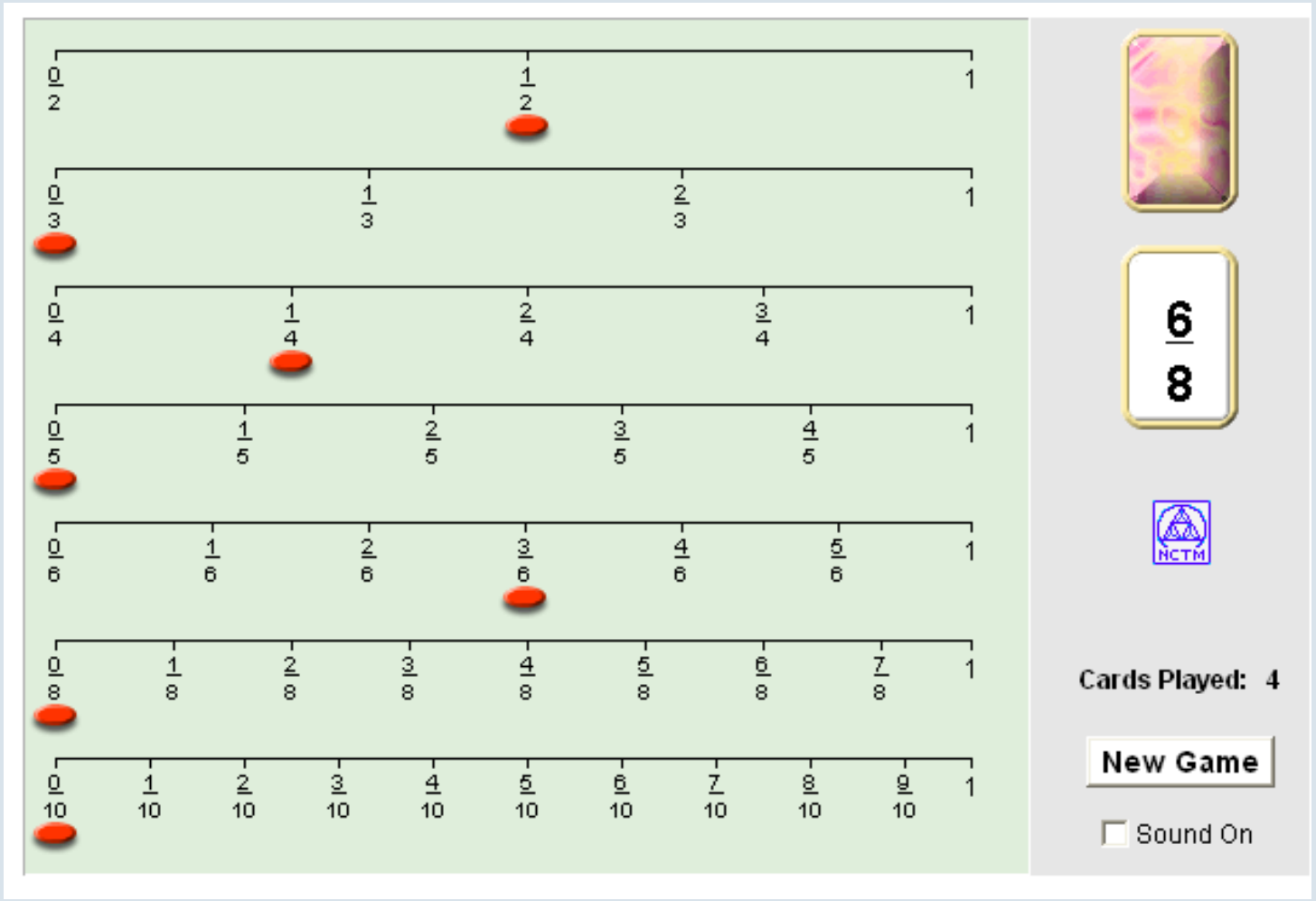
How about if one of the tick marks on the number line was not there? How would students respond?



Write a fraction to show how much of the large square is shaded.



Fraction Game



http://illuminations.nctm.org/ActivityDetail.aspx?ID=18

Now Let's Do Some Math!

Fraction Sort

- Sort the fraction cards

- Fractions equal to or close to 0
- Fractions equal to or close to 1
- Fractions equal to or close to ½

Fraction Estimation

- **Add these fractions in your sort:**

$\frac{5}{9}$ $\frac{1}{12}$ $\frac{17}{20}$

- **Estimate the sum:**

$\frac{4}{5}$ $\frac{7}{12}$

$\frac{1}{8}$ $\frac{8}{9}$

Estimate the answer to (12/13) + (7/8)

A. 1

B. 2

C. 19

D. 21

Only 24% of 13 year olds answered correctly. Equal numbers of students chose the other answers.

NAEP

Fractions in Context

When pitching, Joe struck out 7 of the 12 batters

- **Exactly half = ½**
- **About half = ½**
- **Less then half < ½**
- **More than half > ½**

Sally blocked 5 goals out of 8 attempt

Of the 100 coins in Jim's bank, 34 were pennies

Come up with a few fractions in context with your partner.

Equivalent

- There are many ways to name the same number.
  - 5 can be written as 1 + 4 or 2 + 3.
  - 700 + 80 + 3 can be written as 783.
- There are different names for fractions, too one-third can be written as two-sixths.
  - When two fractions name the same number, we say they are equivalent.

Folding Fractions

Equivalence – many ways to name the same number.

1. Fold a piece of paper into thirds.
2. Unfold, then color one third.
3. Make and record table.

Colored Parts	Total Parts	Fraction Colored
1	3	1/3

4. Now fold the paper in half the other way.
5. Record your results on the table.
6. Explore other possible equivalent fractions.

What other mathematic tools could teachers use to help students build concrete understanding of equivalent fractions?

- Pattern block
- Fraction bars
- Rods
- Number lines

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

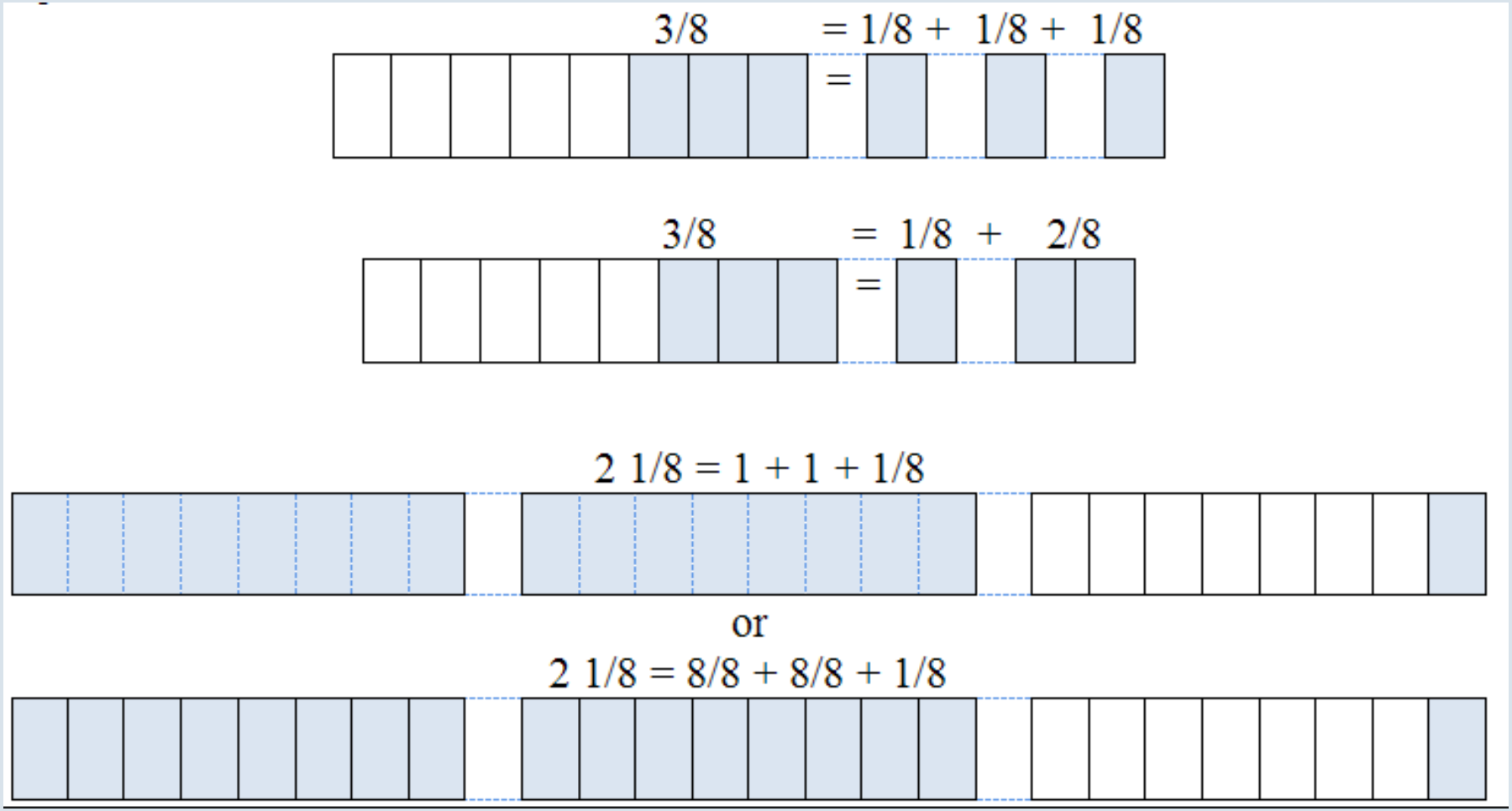
a. Understand addition and subtraction of fractions as **joining and separating parts** referring to the same whole.

b. **Decompose a fraction** into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

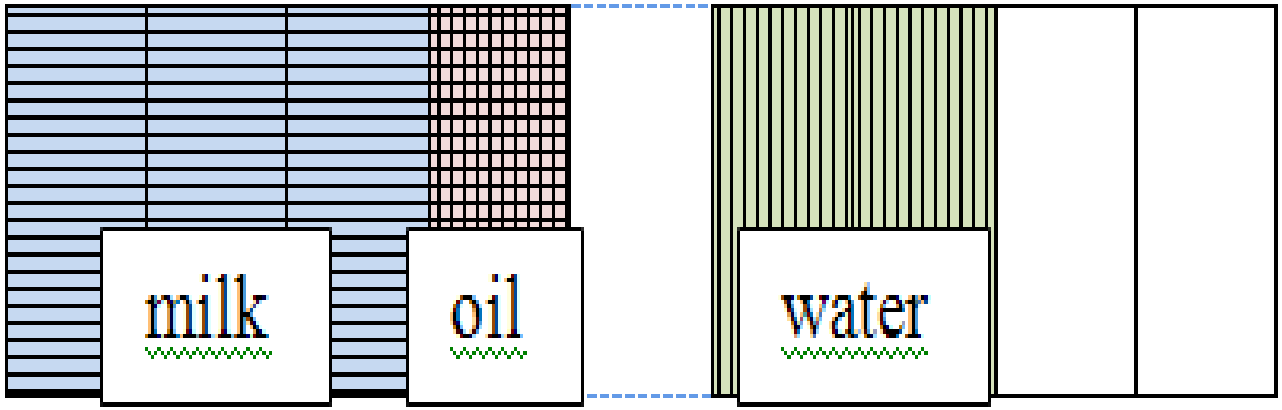
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Decomposing Fractions



Adding Fractions

A cake recipe calls for you to use ¾ cup of milk, ¼ cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?



3/4 + 1/4 + 2/4 = 6/4 = 1 2/4

Fractions on a Number Line

- What does it look like?
- Work with a partner and model the use of a number line when adding and subtracting fractions.
- Share your model with others at your table.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

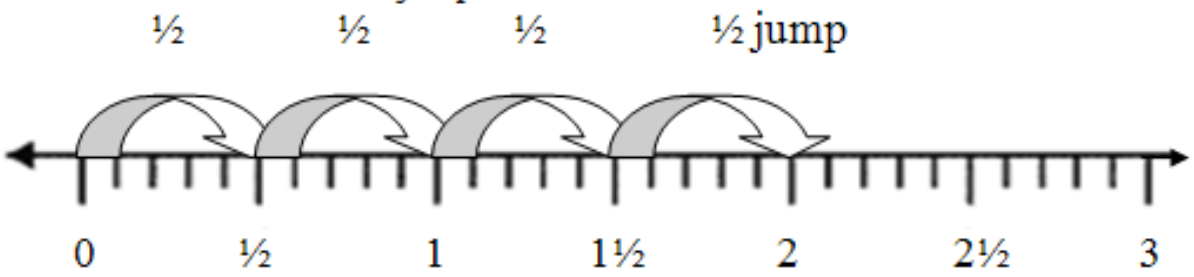
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

Student 1

Draws a number line shows 4 jumps of ½

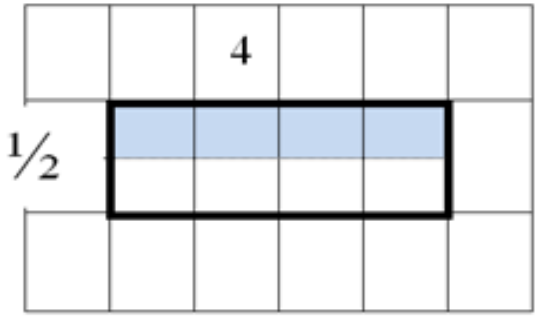


Student 2

Draws an area model showing 4 pieces of ½ joined together to equal 2.

Student 3

Draws an area model representing 4 x ½ on a grid, dividing one row into ½ to represent the multiplier



Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Using the Paper Folding to Multiply Fractions

$$\frac{1}{2} \times \frac{1}{3}$$

A third of the class is wearing green. Half of them are boys. How many boys are wearing green?

Multiplication of Fractions

Two-fifths of the employees at a very large company has Type A blood. If ½ of the company's employees donate blood what fraction will donate type A blood.

Blue = company

Multiplication of Fractions

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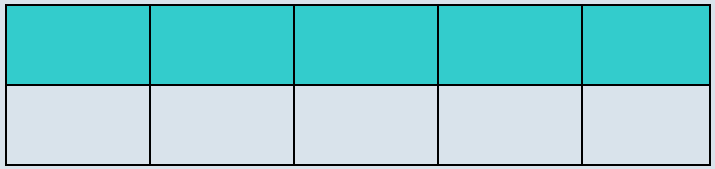
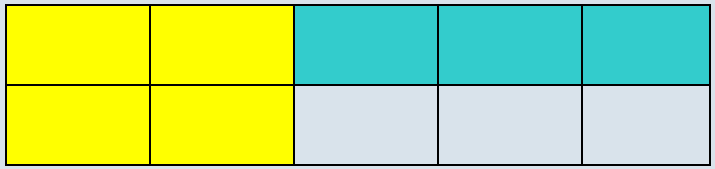


Multiplication of Fractions

Two-fifths of the employees at a very large company has Type A blood. If ½ of the company's employees donate blood what fraction will donate type A blood.

Blue = company

Yellow = Employees with Type A blood



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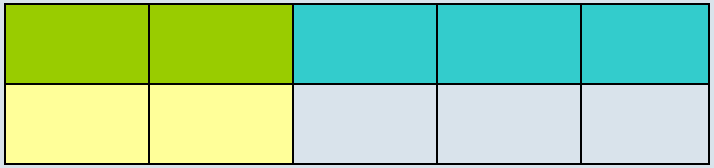
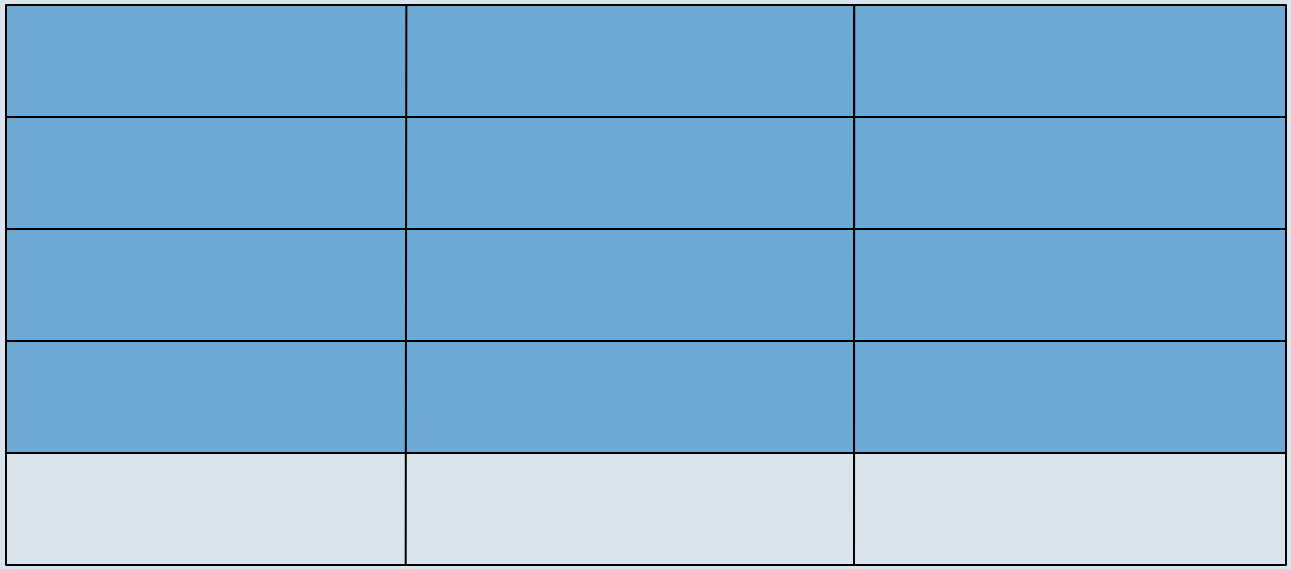


Multiplication of Fractions

$$\frac{2}{3} \cdot \frac{4}{5}$$

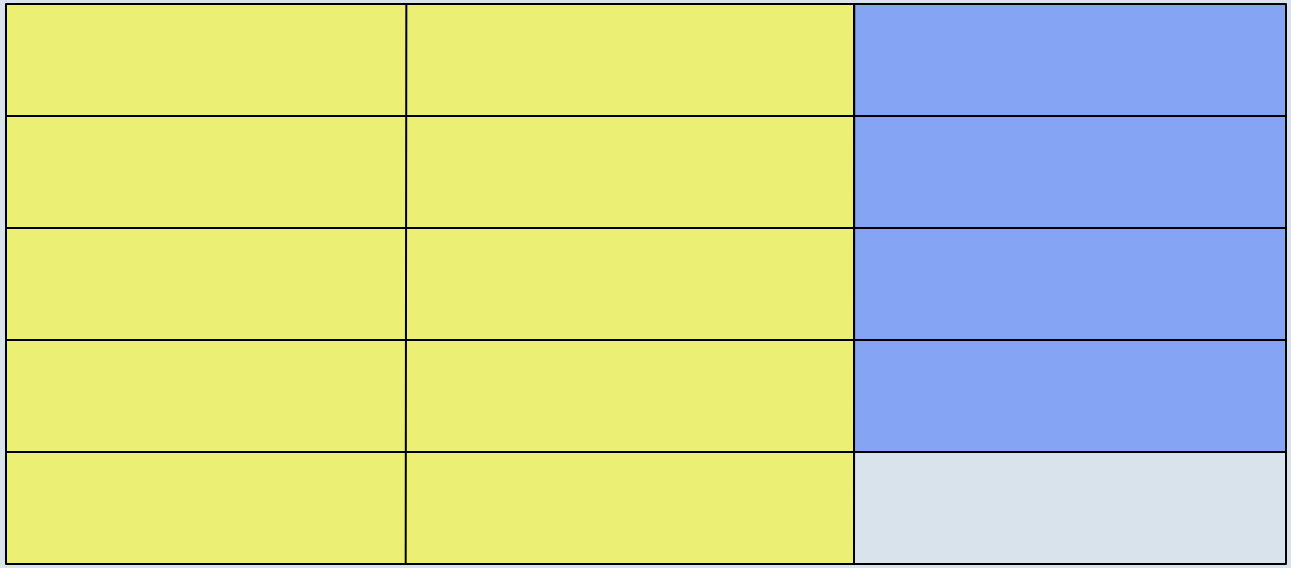
Multiplication of Fractions

$$\frac{2}{3} \cdot \frac{4}{5}$$



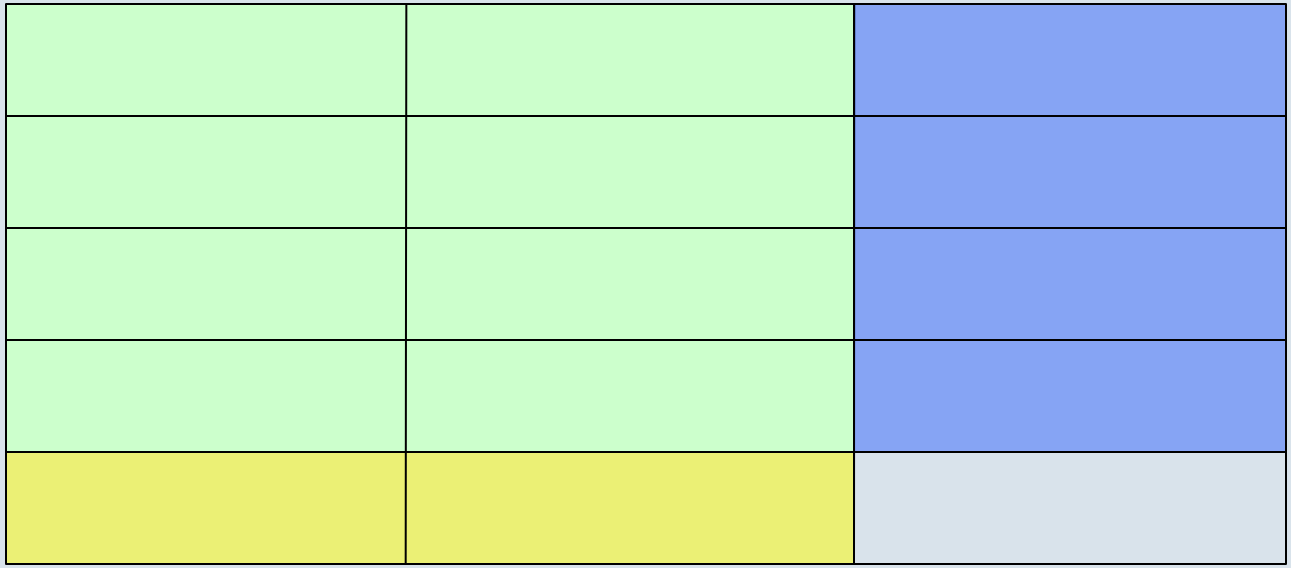
Multiplication of Fractions

$$\frac{2}{3} \cdot \frac{4}{5}$$



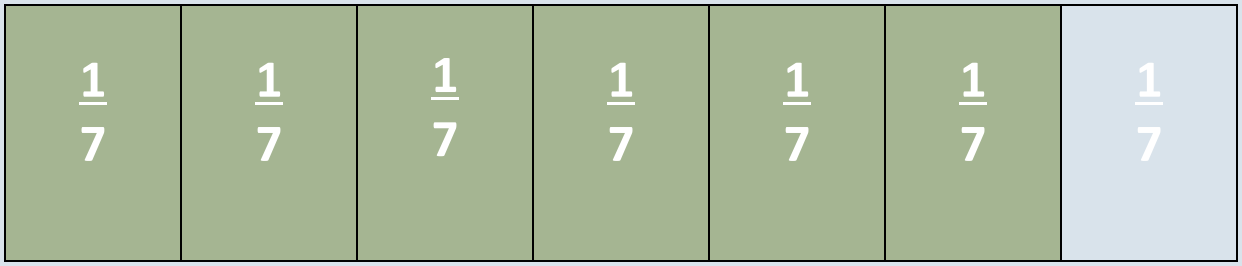
Multiplication of Fractions

$$\frac{2}{3} \cdot \frac{4}{5}$$



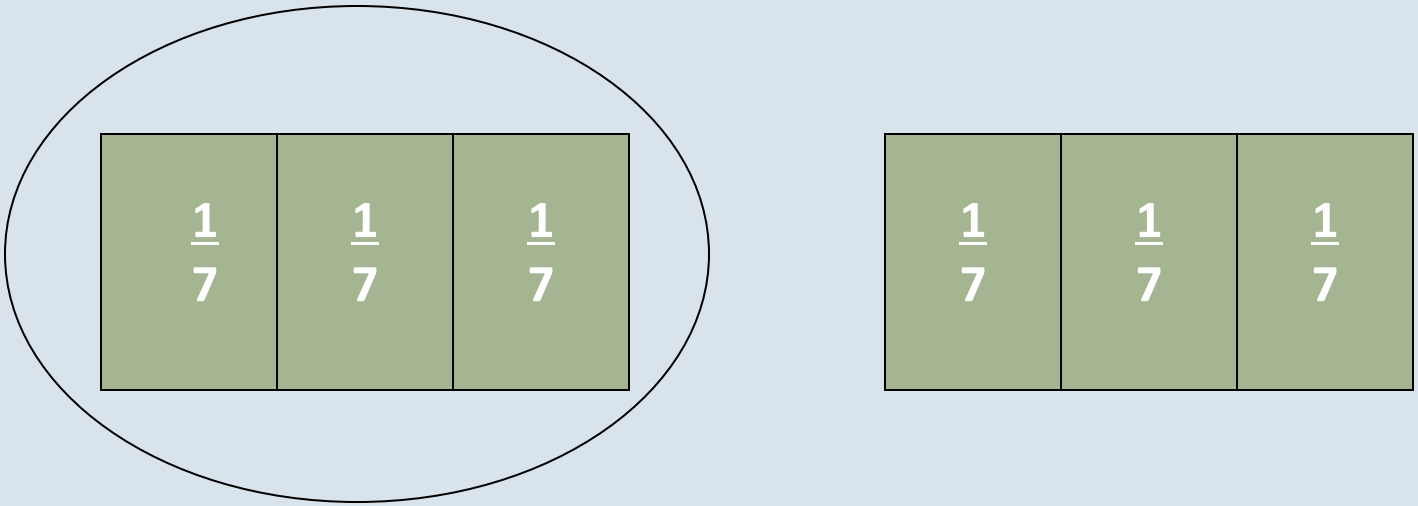
Multiplication of Fractions

$$\frac{1}{2} \cdot \frac{6}{7}$$



Multiplication of Fractions

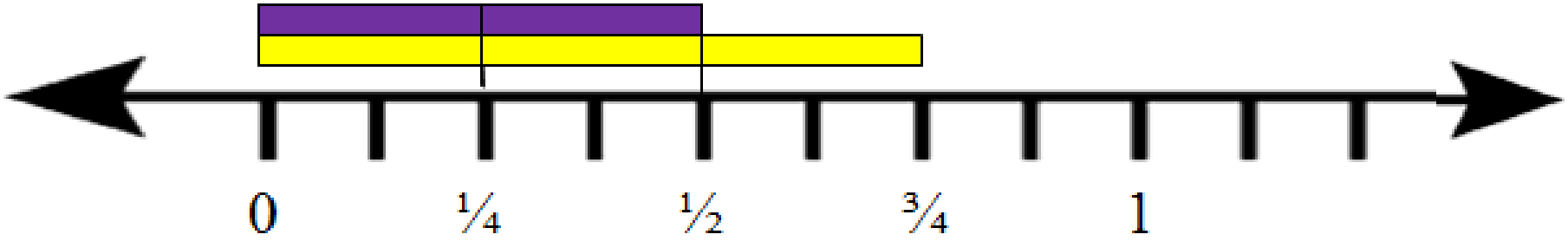
$$\frac{1}{2} \cdot \frac{6}{7}$$



Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

This question is asking what is 2/3 of 3/4
or what is 2/3 x 3/4.

Student 2



5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1

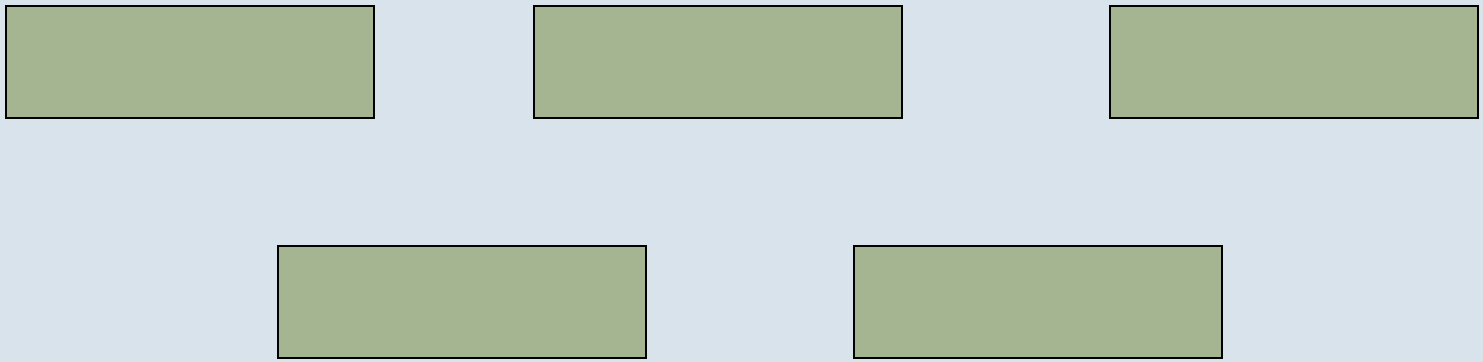
a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

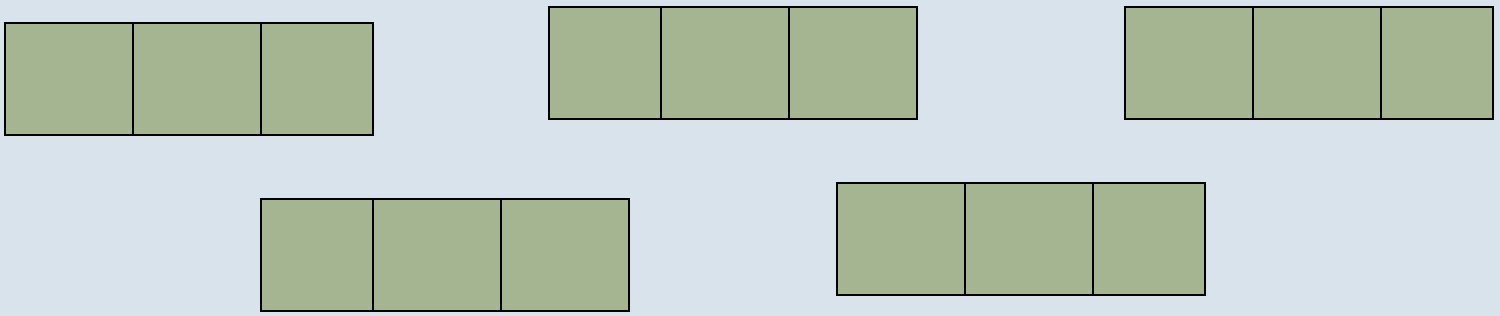
Division of Fractions

$$5 \div \frac{1}{3} = ?$$



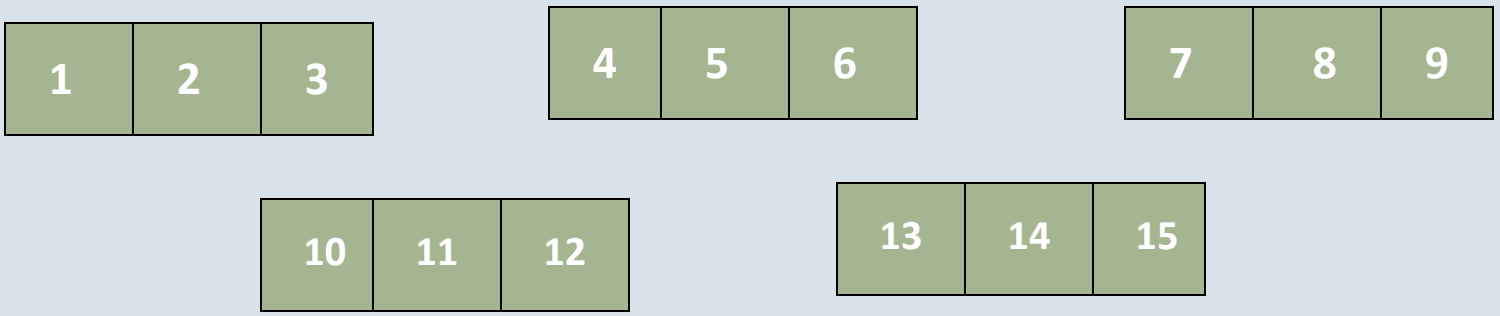
Division of Fractions

$$5 \div \frac{1}{3} =$$



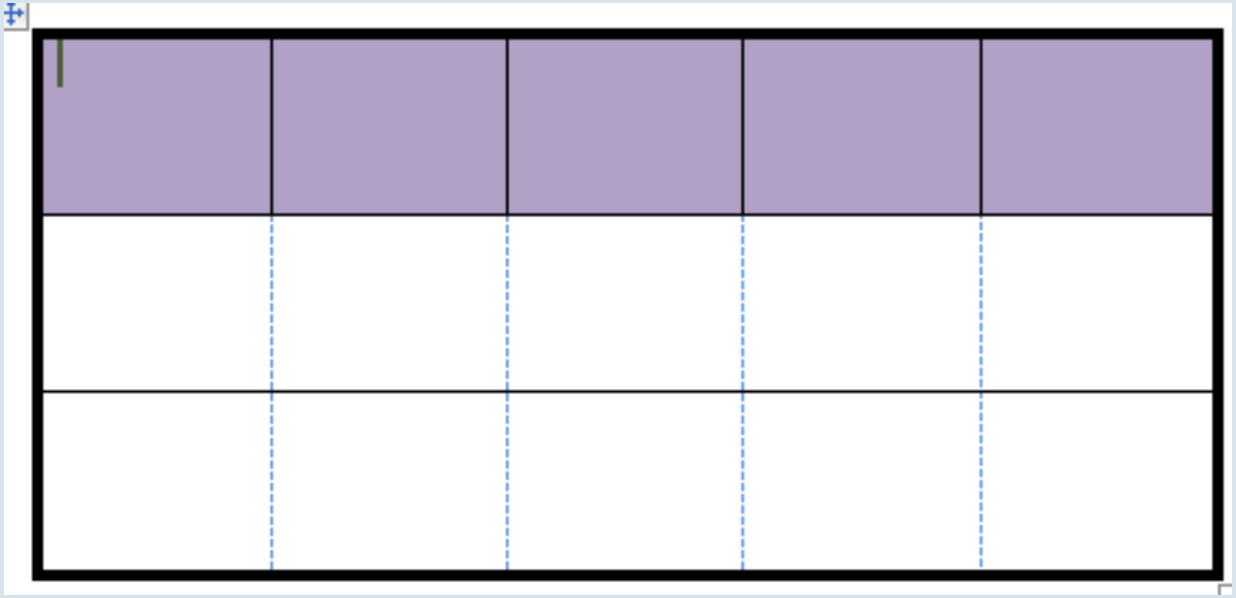
Division of Fractions

$$5 \div \frac{1}{3} =$$



Division of Fractions

$$\frac{1}{3} \div 5 =$$



How is 1/3 ÷ 5 different?

- ***Use the relationship between multiplication and division to explain that (1/3) ÷ 5 = 1/15 because (1/15) × 5 = 1/3.***

- ***Create a story context***

How would this look on a number line?

“The value of the common core is only as good as the implementation of the mathematical practices.”

-- Jere Confrey

Time to Reflect

Summary	


PSST_SUSIE! WHAT'S 12+7?
A BILLION.
THANKS!
WAIT A MINUTE. THAT CAN'T BE RIGHT...
THAT'S WHAT SHE SAID 3+4 WAS.

Questions and Comments

Cloze Reading Activity

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Cloze Reading Activity

Make sense of problems and persevere in solving them.

Mathematically _____ students start by _____ to themselves the meaning of a _____ and looking for _____ points to its solution. They _____ givens, constraints, relationships and goals. They make _____ about the form and meaning of the solution and plan a solution _____ rather than simply jumping into a solution attempt.

Cloze Reading Activity

Make sense of problems and persevere in solving them.

Mathematically _____proficient_____ students start by _____explaining_____ to themselves the meaning of a _____problem_____ and looking for _____entry_____ points to its solution. They _____analyze_____ givens, constraints, relationships and goals. They make _____conjectures_____ about the form and meaning of the solution and plan a solution _____pathway_____ rather than simply jumping into a solution attempt.

Cloze Reading Activity

Construct viable arguments and critique the reasoning of others.

Mathematically _____ students understand and _____ stated assumptions, definitions, and previously established results in constructing _____. They make conjectures and build a logical progression of _____ to explore the _____ of their conjectures.

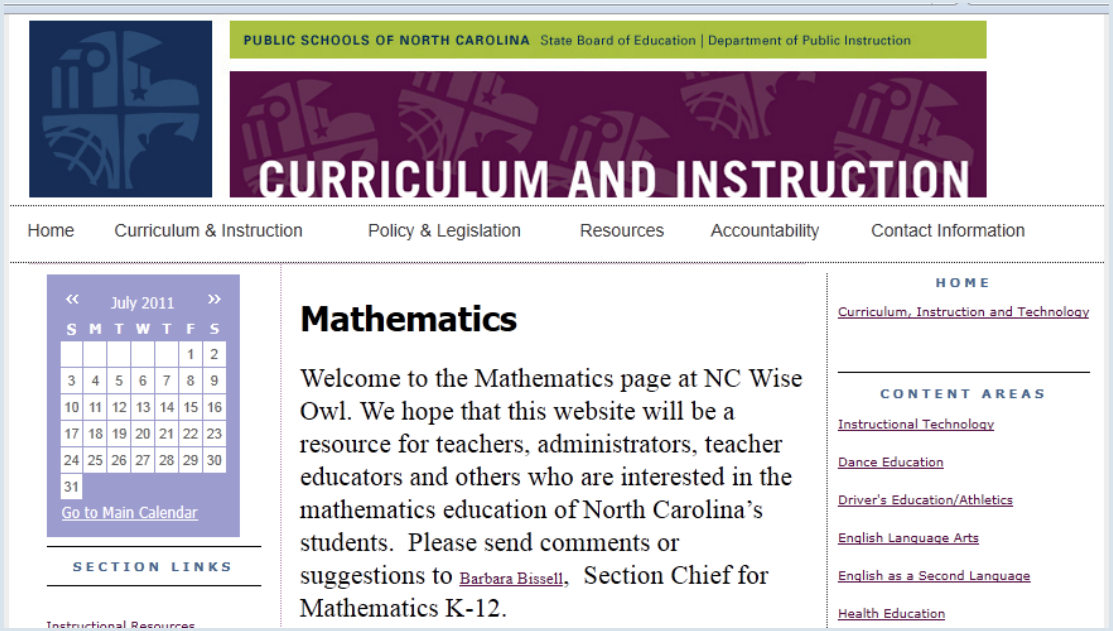
Cloze Reading Activity

Construct viable arguments and critique the reasoning of others.

Mathematically ___proficient___ students understand and ___use___ stated assumptions, definitions, and previously established results in constructing ___arguments___. They make conjectures and build a logical progression of ___statements___ to explore the ___truth___ of their conjectures.

Can’t wait until 2012

DPI Mathematics Site



http://math.ncwiseowl.org

www.corestandards.org

Something to Think About

- What if we didn't have a requirement for math – how would we lure students in?

-- Jere Confrey

Time to Reflect

Summary	

"I think that's right, but let me check."

Questions

Contact Information

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Resources Referenced

- Beyond Pizzas and Pies
  by McNamara and Shaughnessy
- Children's Mathematics, Cognitively Guided Instruction
  by Carpenter, Fennema, Franke, Levi & Empson
- Developing Number Concepts (Books 1-3)
  by Kathy Richardson
- Formative Assessment
  by Margaret Heritage
- Teaching Student-Centered Mathematics, K-3; 3-5
  by Van de Walle and Lovin
- Young Mathematicians at Work
  by Fosnot and Dolk